Received: June 2024, Accepted: September 2024, Published: December 2024
Digital Object Identifier: https://doi.org/10.34302/CJEE/TKYD3692

ENHANCED MULTI-OBJECTIVE GREY WOLF OPTIMIZATION USING ADAPTIVE DIVERSITY TUNING AND LEVY FLIGHTS

Peter **ASIGRI**, Emmanuel Asuming **FRIMPONG,** Emmanuel Kwaku **ANTO**, Daniel **KWEGYIR,** Francis Boafo **EFFAH**
Department of Electrical and Electronic Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana
pasigri@knust.edu.gh, eafrimpong.soe@knust.edu.gh, kwakuantoh@yahoo.com, daniel.kwegyir@knust.edu.gh, fbeffah.coe@knust.edu.gh

Keywords: Multi-objective Optimization, Grey Wolf Optimizer, Adaptive Population Diversity, Levy Flight

Abstract*: This study proposes an enhanced Multi-Objective Grey Wolf Optimizer (MOGWO) using adaptive population diversity tuning and levy flight theories (EMOGWO-ADTLF). It addresses the issues of parameter tunning by balancing exploration and exploitation. Using MATLAB and Python library Pymoo, the study implemented and evaluated the performance of EMOGWO-ADTLF using multi-objective test problems. The results were compared to high-performing algorithms like MOGWO, Non-Dominated Sorting Grey Wolf Optimizer (NSGWO), Dynamic Chaos MOGWO (DCMOGWO), Multi-Objective Mayfly Algorithm (MMA), Multi-Objective Antlion Algorithm (MOALO) and Multi-Objective Dragonfly Algorithm (MODA). In this work, inverted generational distance (IGD) and hypervolume (HV) were the metrics used to measure the performance of algorithms. The metrics measure the diversity, coverage, and spread of solutions. The results obtained showed the potency of EMOGWO-ADTLF in approximating the Pareto fronts. It ranks first in overall average scores in IGD and HV, with total rank scores of 17 and 18, respectively.*

1. INTRODUCTION

Multi-objective optimization has become important in solving problems that involve conflicting objectives in the era of computational intelligence [1,2]. In multi-objective optimization, the goal is always to find a set of solutions, each with a different trademark among the objectives. This is unlike single-objective optimization, which gives a single solution [1]. Multi-objective optimizers have been applied in real-world applications. They are found in engineering designs [2,3], environmental management, and financial planning. For example, one can optimize between service quality and energy use in cloud manufacturing [2]. In power systems engineering, there can be a trade-off between technical and economic effectiveness [3–7]. Also in industrial engineering, multi-objective optimizers help to balance conflicting objectives like cost efficiency and process quality [8].

The advantage of multi-objective optimization is its ability to give diverse options for decision-making. This diversity is crucial because it helps us make better decisions, especially when focusing on just one solution could lead to poor and suboptimal results [1,9]. Multi-objective optimizers are of different kinds. Nature-inspired type is widely used due to its ability to navigate complex solutions effectively. One of these algorithms is MOGWO. MOGWO is an optimizer that solves complex multi-objective problems and is easy to implement [1]. This algorithm has successfully been used in many engineering studies including multi-objective power flow studies [10], multi-robot exploration [11], wind speed forecasting [12] and optimal sizing of microgrids [13]. It has also been used to solve problems such as energy planning for smart homes [14] and reactive power dispatch [15] and transportation location routing [16]. Despite the numerous applications of MOGWO, it still has some deficiencies, just like other metaheuristic algorithms.

Common deficiencies of the MOGWO algorithm include limited performance and scalability, especially when the problem has many objectives [1,17]. The challenge of parameter tuning in Grey Wolf Optimizer (GWO) algorithms also exists [18]. The MOGWO algorithm depends on two parameters to balance between exploration and exploitation in solving multi-objective problems, and the choice of these parameters often affects the quality of the solution [19]. Another common issue with MOGWO is local optima entrapment, especially in cases where there is a need to find global optimum from many local optima [2,20–22].

Some improvements have been made to enhance the performance of the MOGWO algorithm. Yang et al. [23] proposed an improved MOGWO using the ranked-order-value rule for dynamic archive maintenance and solution representation. This algorithm performed better in coverage, spread, and convergence than MOGWO and Multi-Objective Particle Swarm Optimizer (MOPSO). Using a backward learning strategy, Yang et al [2] improved MOGWO to address diversity and local optimum issues. Tian et al. [20] improved MOGWO using multiple techniques. The strategy involved clustering non-dominated solutions, cluster density head wolves' selection, and mutation operator for improved exploration. The results showed an enhanced distribution and diversity compared to the MOGWO algorithm. Another work by Gu [21] introduced an improved MOGWO (DCMOGWO) using dynamic chaos search techniques to solve local optima issues and search precision. DMOGWO outperformed

MOGWO and other nature-inspired algorithms in benchmarked problems. Tlili et al. [17] developed an improved MOGWO (IMOGWO) to help deal with many objectives. In a comparative study with MOGWO and other optimizers, IMOGWO provided better convergence and exploration. Al-Tashi et al. [24] proposed a binary variant of MOGWO (BMOGWO-S) to enhance feature selection in classification. BMOGW-S showed effective classification and feature reduction error rates compared with multi-objective optimizers using the UCI dataset. Other variants of MOGWO that offer enhanced performance include NSGWO [25], Levy-based MOGWO (LMOGWO) [26], improved MOGWO based on individual diversity (IMOGWO) [27], Advanced MOGWO (MOAGWO) [28] and Hybrid MOGWO (HMOGWO) [29].

The aforementioned variants of MOGWO have advanced its performance. However, fundamental defects like avoiding local optima, enhanced parameter tuning, maintaining diversity, and improving convergence still need attention. Though some variants introduced techniques and mechanisms for enhanced parameter tuning, the issue of tuning parameters to balance between exploration and exploitation has not been comprehensively tackled. There is still the need for intuitive and efficient approaches to parameter tuning.

This study proposes an Enhanced MOGWO using adaptive diversity tuning and levy flight (EMOGWO ADTLF) theories to enhance parameter tuning. This enhancement adjusts the control parameters dynamically to balance between exploration and exploitation. This ensures better exploration by reaching the global optimal solutions and reducing local optima entrapment. It also provides better convergence of the obtained Pareto solution and robustness in handling complex and different optimization tasks.

The rest of the paper is structured into sections: Section 2 explains the MOGWO. Section 3 presents the proposed EMOGWO-ADTLF using the population diversity tuning technique and levy flight theories. Section 4 highlights the benchmark functions used for testing and test parameters. Section 5 presents the results of implementing the enhanced MOGWO and its comparison with others. Conclusions drawn are provided in section 6.

2. MULTI-OBJECTIVE GREY WOLF OPTIMIZER

2.1. MOGWO Algorithm

MOGWO is a nature-inspired algorithm based on the hunting behavior of grey wolves [1]. This algorithm advances the GWO, which can only solve a single objective problem [30].

The GWO employs simulated social leadership and encircling behavior to discover optimal solutions. With regard to social leadership, the decreasing order of dominance is designated as alpha (α), beta (β), delta (δ) and omega (ω) wolves. This hierarchy influences the decision-making process in the search space [1,30].

The encircling mechanism observed in grey wolves during hunting is modeled using (1) and (2).

$$\vec{D} = \left|\vec{C} . \vec{X}_p(t) - \vec{X}(t)\right| \tag{1}$$

$$\vec{X}(t + 1) = \vec{X}_p (t) - \vec{A} \cdot \vec{D} \tag{2}$$

$\vec{X}_p$ is the location vector of the prey, while $\vec{X}$ is the position vector of a wolf, and t indicates the current iteration. The vectors $\vec{A}$ and $\vec{C}$ represent coefficients.
The calculation of vectors $\vec{A}$ and $\vec{C}$ are given in (3) and (4).

$$\vec{A} = 2\,\vec{a} \cdot \vec{r}_1 - \vec{a} \tag{3}$$

$$\vec{C} = 2 \cdot \vec{r}_2 \tag{4}$$

Here, $\vec{a}$ (convergence factor) linearly decreases from 2 to 0 during the iterations. $\vec{r}_1$ and $\vec{r}_2$ are random vectors within the [0, 1] range.

The GWO preserves the top three solutions obtained thus far and compels other search agents, including the ω, to adjust their locations relative to these solutions. Equations (5) to (11) are iteratively applied to each search agent throughout the search process, simulating the hunting behavior and identifying promising areas within the search region [30].

$$\overrightarrow{D_\alpha} = \left|\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}\right| \tag{5}$$

$$\vec{D}_\beta = \left|\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}\right| \tag{6}$$

$$\vec{D}_\delta = \left|\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}\right| \tag{7}$$

$$\vec{X}_1 = \left|\vec{X}_\alpha - \vec{A}_1 \cdot (\vec{D}_\alpha)\right| \tag{8}$$

$$\vec{X}_2 = \left|\vec{X}_\beta - \vec{A}_2 \cdot (\vec{D}_\beta)\right| \tag{9}$$

$$\vec{X}_3 = \left|\vec{X}_\delta - \vec{A}_3 \cdot (\vec{D}_\delta)\right| \tag{10}$$

$$\vec{X}(t + 1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{11}$$

The GWO optimization process begins by randomly generating solutions as the initial population. Through the optimization, the three best solutions obtained thus far are saved and designated as α, β, and δ) solutions. The position updating equations (5) to (11) are activated for each wolf (search agents excluding alpha, beta, and delta). Simultaneously, parameters $\vec{A}$ and $\vec{a}$ experience a linear decrease over the iteration. Consequently, search agents move away from the prey when $\vec{A}$>1 and move close to the prey when $\vec{A}$<1. Ultimately, the position and score of the α solution are recorded as the best solutions achieved during optimization once the iterations have ended [30].

To turn the GWO into a multi-objective optimizer (MOGWO), two features are added. The first component is the archive, and the second is the leader selection strategy. The archive stores the non-dominated solutions. The leader selection strategy helps obtain the α, β, and δ solutions and make them leaders. In the archive, an archive controller manages, saves, and retrieves Pareto solutions during iterations. A specific rule governs the entry of new solutions into the archive. If an archive member dominates any new solution, entry is not allowed. New solutions can enter the archive if it dominates one or more members. In this case, the dominated solutions are deleted. New solutions are also allowed entry if there is no dominance between them and stored archive solutions. The MOGWO algorithm has a grid mechanism that helps to rearrange the objective space when an archive gets full. The mechanism deletes solutions from the most crowded areas and stores the new solutions in the least overcrowded zones. This ensures better distribution of solutions [1].

GWO algorithm uses the parameter $'\boldsymbol{a}'$ to decide the search radius. This value controls the exploration-exploitation trade-off. The parameter ranges from 0 to 2, decreasing linearly during the iteration. This decrease assists the algorithm by reducing parameter tuning. However, some defects may affect the algorithm performance depending on the problem [19]. The challenges include:

- Excessive early exploration: If the initial value of $'\boldsymbol{a}'$ is too high, the algorithm will likely over-explore without looking for optimal regions. This may delay convergence.
- Premature Exploitation: If $'\boldsymbol{a}'$ decreases rapidly, it could cause early exploitation. The algorithm gets trapped in local optima and misses better solutions.
- Fine-tuning difficulty: Determining the optimal starting and ending value for linear tuning techniques may require adjustment and experimentation depending on the type of problem.

2.2. Proposed EMOGWO-ADTLF Using Population Diversity Tuning Technique and Levy Flight

In this work, population diversity is employed to improve the performance of MOGWO. Population diversity in metaheuristics refers to the variety and spread of possible solutions within the population evaluated by the algorithm. It measures how different the individuals (solutions) in the population are from each other. High diversity means the solutions are spread across a wide area of the search space, while low diversity indicates that solutions are clustered closely together. Maintaining diversity within the population in algorithms is essential in balancing exploration and exploitation. Population diversity reduces premature convergence, helping to prevent suboptimal performance. Diversity is crucial in dynamic optimization problems where the nature of the problem keeps changing. In multi-objective optimization, diversity helps to search the entire Pareto front to determine the global solution [31].

An adaptive population diversity scheme is introduced into MOGWO to deal with the issues of excessive exploration, early exploitation, and fine-tuning difficulty. The proposed adaptive population diversity scheme dynamically adjusts the parameter '$\boldsymbol{a}$' depending on the problem. The proposed scheme is presented in presented in **algorithm 1**.

Algorithm 1: Proposed adaptive scheme to tune '$\boldsymbol{a}$'

start

1 ***Set*** *diversity threshold*

2 ***For*** *each pair of wolves (i, j), calculate the Euclidean distance between their current positions in the solution space using:*

$$distance_{ij} = \left\| X_i - X_j \right\|$$

X_i and X_j are adjacent search agents (wolves).

3 *For each distance calculated, normalize the distances using:*

$$distances = \frac{distances}{max\, distances}$$

4 *Calculate the average of all normalized distances as a measure of the diversity of the wolves using:*

$$diversity = \frac{1}{N(X) \times (N(X) - 1)} \sum_{i=i}^{N(X)-1} \sum_{j=i+1}^{N(x)} distances\ (i, j)$$

where N(X) is the number of wolves.

5 *If diversity < diversity threshold*

6 *Adjust a as;* $a = a \times 1.05$ *(exploration)*

7 *If diversity > diversity threshold*

8 *Adjust a as;* $a = a \times 0.95$ *(exploitation)*

9 *Adjust a to stay within bounds as;* $\max[0.1, \min(a, 2)]$

10 ***End***

11 ***End***

The adaptive scheme enhances MOGWO in the following ways.

- It adjusts $'\boldsymbol{a}'$ upward to allow the search agents (wolves) to explore new optimal regions by moving away from their current location when there is high level of similarity within the population.
- It adjusts $'\boldsymbol{a}'$ downwards to allow search agents to focus narrowly on richer regions when solutions are highly scattered. This encourages exploitation and enhances convergence to the optimal solution.

The adaptive technique provides flexible tuning of parameters. It not only improves convergence but also provides robustness. The scheme improves convergence because stagnation and excessive exploration are prevented. The tuning method equips MOGWO with the robustness to effectively deal with complex and diverse problems.

In addition to the adaptive scheme, a Levy flight operator is employed to enhance the algorithm. Levy flight is a random walk with a step size that follows a heavy-tailed probability distribution. This approach is used in optimization algorithms to enable a search strategy that combines local and global exploration efficiently [32]. Using steps of varying lengths, levy flight helps the algorithm explore better by reaching diverse regions and escaping local optima entrapment [33].

To implement the levy-based technique in MOGWO, the levy flight operator is introduced into (4) to modify the parameter $'\boldsymbol{C}'$. In (4), $'\boldsymbol{C}'$ is a critical parameter determining the quality of solution updates in the GWO.
The modified $'\boldsymbol{C}'$ parameter is defined according to (12)

$$\vec{C} = levy\ number\ (dependent\ on\ number\ of\ search\ agents) \tag{12}$$

Levy Flight operator is applied to equation (4) to improve MOGWO as follows.

Step 1: Calculate the step size (Δ) for the levy distribution using (13). This ensures that the step size is appropriate for the problem's dimensionality.

$$\Delta = \frac{1}{\sqrt{D}} \tag{13}$$

where D is the problem's dimension.

Step 2: Generate a random number from the Cauchy distribution as a Cauchy number (f(x)). This work uses the standard probability distribution function (PDF) with x having location parameter 0 and scale parameter 1 defined according to (14).

$$f(x) = \frac{1}{\pi(1+x^2)} \tag{14}$$

Step 3: Calculate the levy number (L) using (15).

$$L = s \times f(x) \tag{15}$$

The flow chart of EMOGWO-ADTLF incorporating adaptive population diversity and levy flight is shown in *fig. 1*.

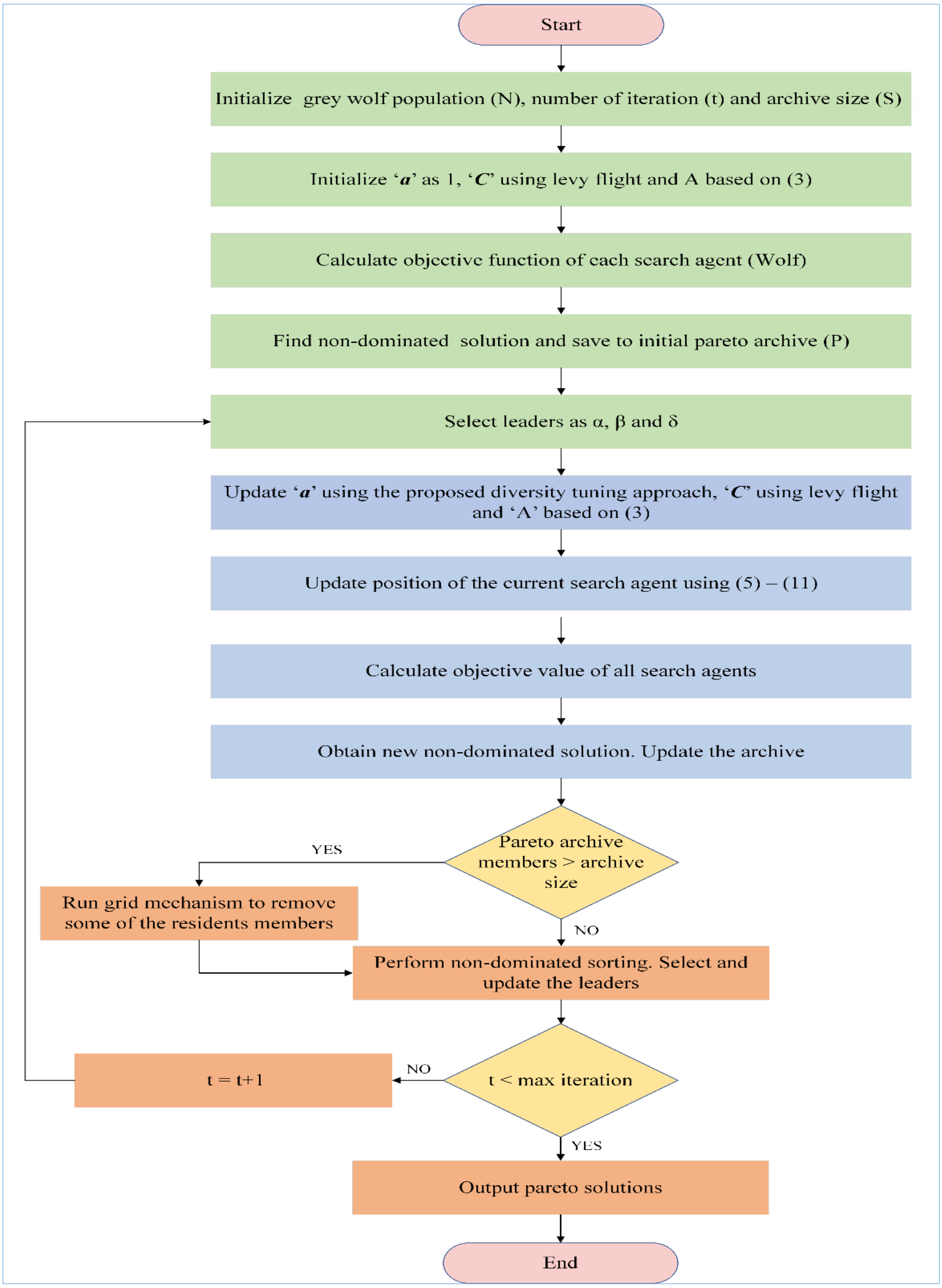

Fig. 1. Flow Chart of EMOGWO-ADTLF

3. BENCHMARK FUNCTIONS USED AND TEST PARAMETERS

3.1. Benchmark Functions

Eight standard multi-objective test functions in the CEC 2009 [34] were used as test beds. Table 1 presents the benchmark problems chosen. These test functions offer diverse multi-objective search spaces with distinct Pareto fronts, including convex, non-convex, discontinuous, and multi-modal scenarios.

In addition, this study also used two benchmark real-world engineering problems. The use of these problems demonstrates the applicability and robustness of multiobjective optimization algorithms in solving complex, real-world engineering tasks that involve multiple conflicting objectives. They include the design of the welded beam and the Disc Brake. The welded beam multi-objective design is a well-known test problem in many studies. This benchmark design has four variables. They include the beam’s thickness (h), width (w), depth (d) and length (L). The objective is to reduce the fabrication cost (c) and the end deflection (δ). The main constraints are shear stress, bending stress, and buckling load [35,36]. The detailed equations of this problem are provided in Table 2.

The goals of the multiple-disc brake are to reduce the brake's mass and minimize the stopping time. The variables to be determined in the design are the force (F), the number of friction surfaces (s) as well as inner and outer radius (r and R). The design must adhere to several constraints, including the minimum allowable length between the radii, the maximum allowable length of the brake, as well as limitations related to pressure, temperature, and torque [35,36]. The objectives and constraints of this problem are also shown in Table 2.

Table 1. Test Functions, UF1 – UF8

Function	Mathematical Expression
UF1	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}\left[x_j - sin\left(6\pi x_1 + \frac{j\pi}{n}\right)\right]^2$, $f_2 = 1 - \sqrt{x} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}\left[x_j - sin\left(6\pi x_1 + \frac{j\pi}{n}\right)\right]^2$, $J_1 = \{j \mid j \text{ is odd and } 2 \le j \le n\}$, $J_2 = \{j \mid j \text{ is even and } 2 \le j \le n\}$ $PF\ f_2 = 1 - \sqrt{f_1}, \quad 0 \le f_1 \le 1$
UF2	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} {y_j}^2$: $f_2 = 1 - \sqrt{x} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_1} {y_j}^2$, $J_1 = \{j \mid j \text{ is odd and } 2 \le j \le n\}$, $J_2 = \{j \mid j \text{ is even and } 2 \le j \le n\}$, y_j $= \begin{cases} x_j - \left[0.3{x_1}^2\cos\left(24\pi x_1 + \frac{4j\pi}{n}\right) + 0.6x_1\right]\cos\left(6\pi x_1 + \frac{j\pi}{n}\right) \text{ if } j \in J_1 \\ x_j - \left[0.3{x_1}^2\cos\left(24\pi x_1 + \frac{4j\pi}{n}\right) + 0.6x_1\right]\cos\left(6\pi x_1 + \frac{j\pi}{n}\right) \text{ if } j \in J_2 \end{cases}$ $PF: f_2 = 1 - \sqrt{f_1}, \quad 0 \le f_1 \le 1$

Function	Mathematical Expression
UF3	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\left(4\sum_{j\in J_1} {y_j}^2 - 2\prod_{j\in J_1}\cos\left(\frac{20y_j\pi}{\sqrt{j}}\right)+2\right) PF: 0 \le f_1 \le 1.$ $f_2 = \sqrt{x_1} + \frac{2}{\lvert J_1\rvert}\left(4\sum_{j\in J_1} {y_j}^2 - 2\prod_{j\in J_1}\cos\left(\frac{20y_j\pi}{\sqrt{j}}\right)+2\right) PF: f_2 = 1-\sqrt{f_1}.$ $J_1 \, and \, J_2 = UF1, \quad y_j = x_j - {x_j}^{0.5\left(1+\frac{3(j-2)}{n-2}\right)}, \quad j = 2,3,\dots,n$,,
UF4	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} h(y_j) \quad f_2 = 1 - x_2 + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} h(y_j), J_1 \, and \, J_2 = UF1,$ $y_j = x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right), \quad j = 2,3,\dots,n, h(t) = \frac{\lvert t\rvert}{1+e^{2\lvert t\rvert}}$ $PF: f_2 = 1 - f^2, \quad 0 \le f_1 \le 1$
UF5	$f_1 = x_1 + \left(\frac{1}{2N}\right) + \epsilon \lvert\sin(2N\pi x_1)\rvert + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} h(y_j),$ $f_2 = 1 - x_1 + \left(\frac{1}{2N}\right) + \epsilon \lvert\sin(2N\pi x_1)\rvert + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} h(y_j),$ $J_1 \, and \, J_2 = UF1, \epsilon > 0 \; y_j = x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right), \quad j = 2,3,\dots,n, h(t)$ $= 2t^2 - \cos(4\pi t) + 1$ Its PF has $2N+1$ solutions: $\left(\frac{i}{2N}, 1-\frac{i}{2N}\right)$ for $i = 0,1,\dots,2N$
UF6	$f_1 = x_1 + max\left\{0, 2\left(\frac{1}{2N} + \epsilon\right)\sin(2N\pi x_1)\right\} + \frac{2}{\lvert J_1\rvert}\left(4\sum_{j\in J_1} {y_j}^2 - \right.$ $\left. 2\prod_{j\in J_1}\cos\left(\frac{20y_j\pi}{\sqrt{j}}\right)+1\right)$ $f_2 = 1 - x_1 + max\left\{0, 2\left(\frac{1}{2N} + \epsilon\right)\sin(2N\pi x_1)\right\} + \frac{2}{\lvert J_2\rvert}\left(4\sum_{j\in J_2} {y_j}^2 - \right.$ $\left. 2\prod_{j\in J_2}\cos\left(\frac{20y_j\pi}{\sqrt{j}}\right)+1\right)$ $J_1 \, and \, J_2 = UF1, \; y_j = x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right), \quad j = 2,3,\dots,n$ *PF: One isolated* $(0,1)$, *and N disconnected:* $f_2 = 1 - f_1, f_1 \in$ $\cup_{i=1}^{N}\left[\frac{2i-1}{2N}, \frac{2i}{2N}\right]. N = 2$
UF7	$f_1 = \sqrt[5]{x_1} + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} {y_j}^2 \quad f_2 = 1 - \sqrt[5]{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} {y_j}^2 \quad 0 \le f_1 \le 1$ $J_1 \, and \, J_2 = UF1 \; y_j = x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right), \quad j = 2,3,\dots,n, PF: f_2 = 1 - f_1,$
UF8	$f_1 = \cos(0.5x_1\pi)\cos(0.5x_2\pi) + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}\left(x_j - 2x_2\sin\left(2\pi x_1 + \frac{j\pi}{n}\right)^2\right)$ $f_2 = \cos(0.5x_1\pi)\sin(0.5x_2\pi) + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}\left(x_j - 2x_2\sin\left(2\pi x_1 + \frac{j\pi}{n}\right)^2\right)$ $f_3 = \sin(0.5x_1\pi) + \frac{2}{\lvert J_3\rvert}\sum_{j\in J_3}\left(x_j - 2x_2\sin\left(2\pi x_1 + \frac{j\pi}{n}\right)^2\right)$ $J_1 = \{j \mid 3 \le j \le n, and \; j-1 \; is \; a \; mmultiple \; of \; 3\}$ $, J_2 = \{j \mid 3 \le j \le n, and \; j-2 \; is \; a \; multiple \; of \; 3\}$ PF is $f_1^2 + f_2^2 + f_3^3 = 1,$ $J_3 = \{j \mid 3 \le j \le n, and \; j \; is \; a \; multiplication \; of \; 3\}$, Its , $0 \le f_1, f_2 f_3 \le 1$

Table 2. Benchmarked Engineering Problems

Problem	Objectives and Constraints
Welded Beam Design	minimise $f_1(\boldsymbol{x}) = 1.10471w^2L + 0.04811dh(14.0 + L)$, $f_2 = \delta$, subject to $g_1(\boldsymbol{x}) = w - h \leq 0, g_2(\boldsymbol{x}) = \delta(\boldsymbol{x}) - 2.5 \times 10^{-1} \leq 0,$ $g_3(\boldsymbol{x}) = \tau(\boldsymbol{x}) - 1.36 \text{ x } 10^4\text{x} \leq 0, g_4(\boldsymbol{x}) = \sigma(\boldsymbol{x}) - 3.0 \text{ x } 10^4 \leq 0,$ $g_5(\boldsymbol{x}) = 0.10471w^2 + 0.04811hd(14 + L) - 5.0 \leq 0, g_6(\boldsymbol{x}) = 1.3 \times 10^{-1} - w \leq 0,$ $g_7(\boldsymbol{x}) = 6{,}000 - P(\boldsymbol{x}) \leq 0, 0.1 \leq L, d \leq 10$ and $1.25 \text{ x } 10^{-1} \leq w, h \leq 2.0$ where $\sigma(\boldsymbol{x}) = \frac{504{,}000}{hd^2}, \quad Q = 6{,}000\left(14 + \frac{L}{2}\right), \text{D} = \frac{1}{2}\sqrt{L^2 + (w + d)^2}$ $J = \sqrt{2}wL\left[\frac{L^2}{6} + \frac{(w + d)^2}{2}\right], \delta = \frac{65{,}856}{30{,}000hd^3}, \quad \beta = \frac{QD}{J}$ $\alpha = \frac{6{,}000}{\sqrt{2}wL}, P = 0.61423 \times 10^6 \frac{dh^3}{6}\left(1 - \frac{d\sqrt{30/48}}{28}\right)$
Brake Disc Design	Minimize $f_1(\boldsymbol{x}) = 4.9 \times 10^{-5}(R^2 - r^2)(s - 1),$ $f_2(\boldsymbol{x}) = \frac{9.82 \times 10^6(R^2 - r^2)}{Fs(R^3 - r^3)}$ subject to $g_1(x) = 20 - (R - r) \leq 0,$ $g_2(x) = 2.5(s + 1) - 30 \leq 0$ $g_3(x) = \frac{F}{3.14(R^2 - r^2)} - 0.4 \leq 0,$ $g_4(\boldsymbol{x}) = \frac{2.22 \times 10^{-3}F(R^3 - r^3)}{(R^2 - r^2)^2} - 1 \leq 0,$ $g_5(\boldsymbol{x}) = 900 - \frac{2.66 \text{ x } 10^{-2}Fs(R^3 - r^3)}{(R^2 - r^2)} \leq 0.$ $5.5 \text{ x } 10 \leq r \leq 8.0 \text{ x } 10, 7.5 \text{ x } 10 \leq R \leq 1.1 \text{ x } 10^2$ $1.0 \text{ x } 10^3 \leq F \leq 3.0 \text{ x } 10^3, 2 \leq s \leq 20.$

3.2. Performance Metrics

This study uses two performance metrics with abilities to test for convergence, diversity, and spread to measure the performance of the EMOGWO-ADTLF and compare it with other muti-objective algorithms. The metrics include Inverted Generational Distance (IGD) [37] and Hypervolume (HV) [38].

IGD is a measue that determines the diversity and convergence of a multi-objective algorithm. It assesses how close the obtained solutions are to the Pareto front. It also determines how the obtained solution spread over the Pareto solution. The value of IGD obtained is an indication of how well an algorithm maintains its balance. A low IGD value shows an algorithm is well balanced. It shows that the algorithm is neither wandering in non-optimal regions (over-exploration) nor stuck in local optima (over-exploitation) [37]. IGD is mathematically represented as (19).

$$IGD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n} \tag{19}$$

where n is the count of pareto solutions, d_i indicates the euclidean distance from the ith pareto optimal solution and the nearest obtained solution.

HV is a performance metric that measures the volume of the Pareto front occupied by the obtained solution. It measures diversity, convergence and spread of the solution. It is a widely used metric for benchmarking the performance of multi-objective optimizers. The HV is determined with respect to a reference point. The reference point is a value worse than the any value in the obtained Pareto solutions. A high hypervolume means a better diversity, convergence, and distribution of the obtained solution. Mathematically, HV is given by (20) [38].

$$HV(S, r) = \lambda_m(\cup_{z \in S} \ [z; r]) \tag{20}$$

λ_m is the Lebesgue measure. It is the size of true Pareto front occupied by the solution. m is the number of objectives. $\cup_{z \in S}$ is the union of points z in the set S. r is the reference point.

3.3. Experimental Setups

The study used two experimental setups. The first experiment compared EMOGWO-ADTLF with MOGWO and two high-performing other variants of MOGWO in the literature namely NSGWO [25] and DCMOGWO [21]. This experiment analyzed the graphs of obtained Pareto against true Pareto and determined IGD and HV values. The second experiment also compared EMOGWO-ADTLF to three well-known, efficient, and robust algorithms. They include Multi-Objective Mayfly Algorithm (MMA) [39] , Multi-Objective Ant Lion Optimizer (MOALO) [36] and Multi-Objective Dragonfly Algorithm (MODA) [40] using multi-objective test functions. In the third experiment, MOGWO was again compared with MMA, MOALO and MODA using real-world engineering benchmarked problems. The metrics of comparison were the IGD and HV.

This work used MATLAB 2021 to execute all algorithms to obtain the Pareto solutions. The general parameters of all algorithms were as follows:

- Number of search agents: 100
- Number of iterations: 3000

- Number of runs: 03
- Archive size: 100.

These parameters were selected to guarantee a thorough and efficient evaluation of the multi-objective algorithms. Utilizing 100 search agents strikes a practical balance, offering enough diversity without excessively taxing computational resources. The selected 3000 iterations ensure the search process is detailed enough to discover and refine multiple solutions, accounting for the complexity and multi-objective nature of the problems. An archive size of 100 maintains a balance between storing a diverse set of Pareto-optimal solutions and managing computational resources. Conducting multiple runs provides statistically significant insights into the performance of the algorithms.

Subsequently, there were comparisons of obtained Pareto solutions and true Pareto fronts using the Python library Pymoo. Pymoo is a specialized multi-objective optimization and analysis tool. The desktop computer used for this study was an HP ZB G4 workstation with the processor Intel® Xeon ® Silver 4108 CPU @ 1.80 GHz (16 CPUs) and memory of 64 GB. This specification provided enough computational power and efficiency for the rigorous simulations and computations in this work.

4. RESULTS AND DISCUSSIONS

4.1. Analysis of EMOGWO-ADTLF against MOGWO and its Variants

Fig. 2 presents the plots of the obtained Pareto fronts against the true Pareto fronts for UF1 to UF3. In the UF1 graph, EMOGEO-ADTFL and MOGWO show better convergence and spread than NSGWO and DCMOGWO. EMOGWO-ADT has a better distribution of Pareto solutions than MOGWO. All algorithms show better convergence, diversity, and distribution in UF2 than in UF1. Of the four algorithms, NSGWO shows better coverage. In UF3, the algorithms struggle to approximate the Pareto fronts with EMOGWO-ADTLF having better spread and convergence than the rest.

Fig. 3 shows the graph of test functions UF4. UF5 and UF6. In UF4, EMOGWO-ADTLF and DCMOGWO closely approximate the Pareto fronts and have better spread. NSGWO and MOGWO show good convergence, but poor distribution compared to EMOGWO-ADTLF and DCMOGWO. All the algorithms find it difficult to approximate the Pareto fronts in UF5 and UF6, with only a few non-dominated Pareto solutions. NSGWO appears to have a better convergence and spread than the rest in UF6.

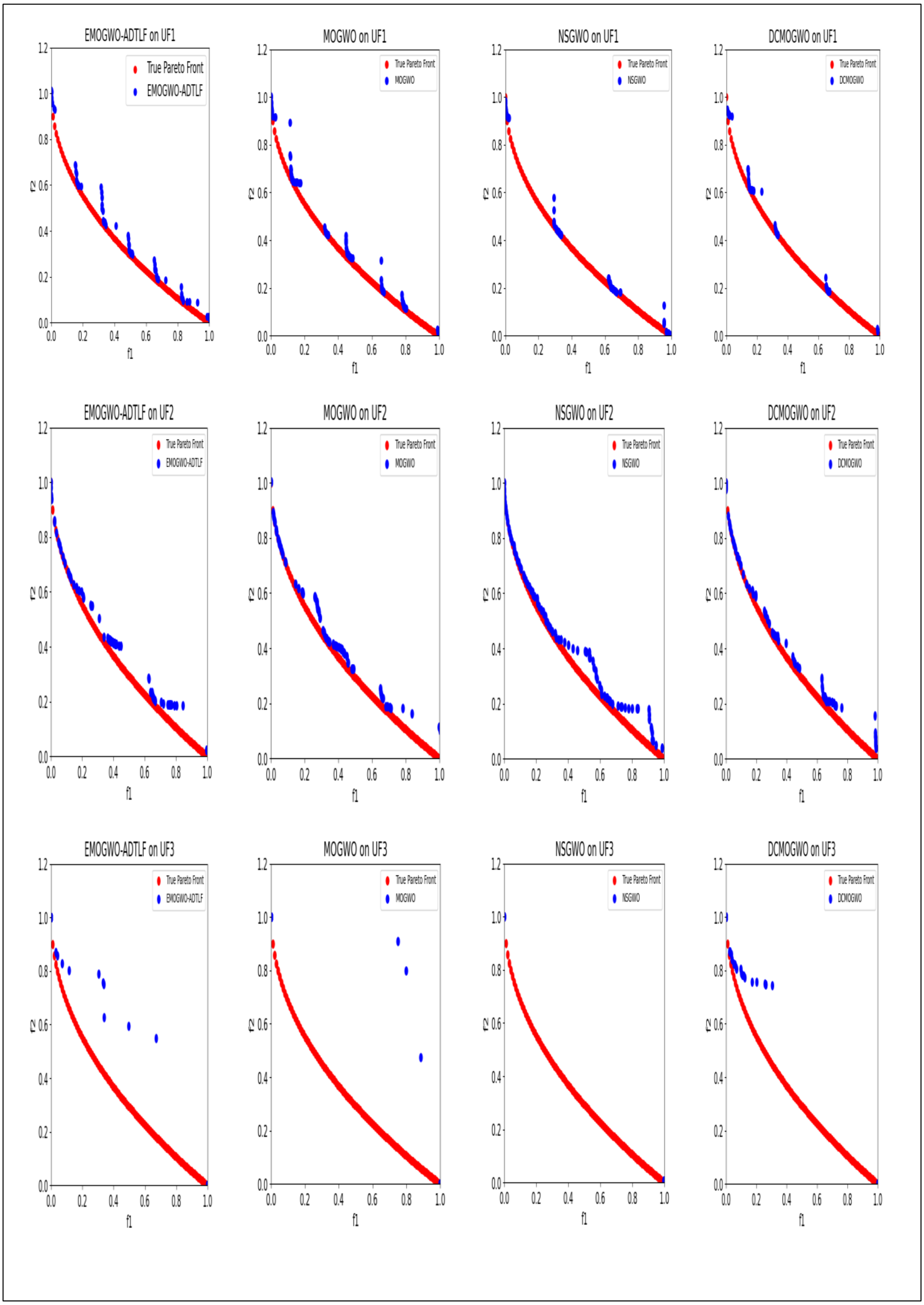

Fig. 2: Graph of test functions UF1, UF2 and UF3

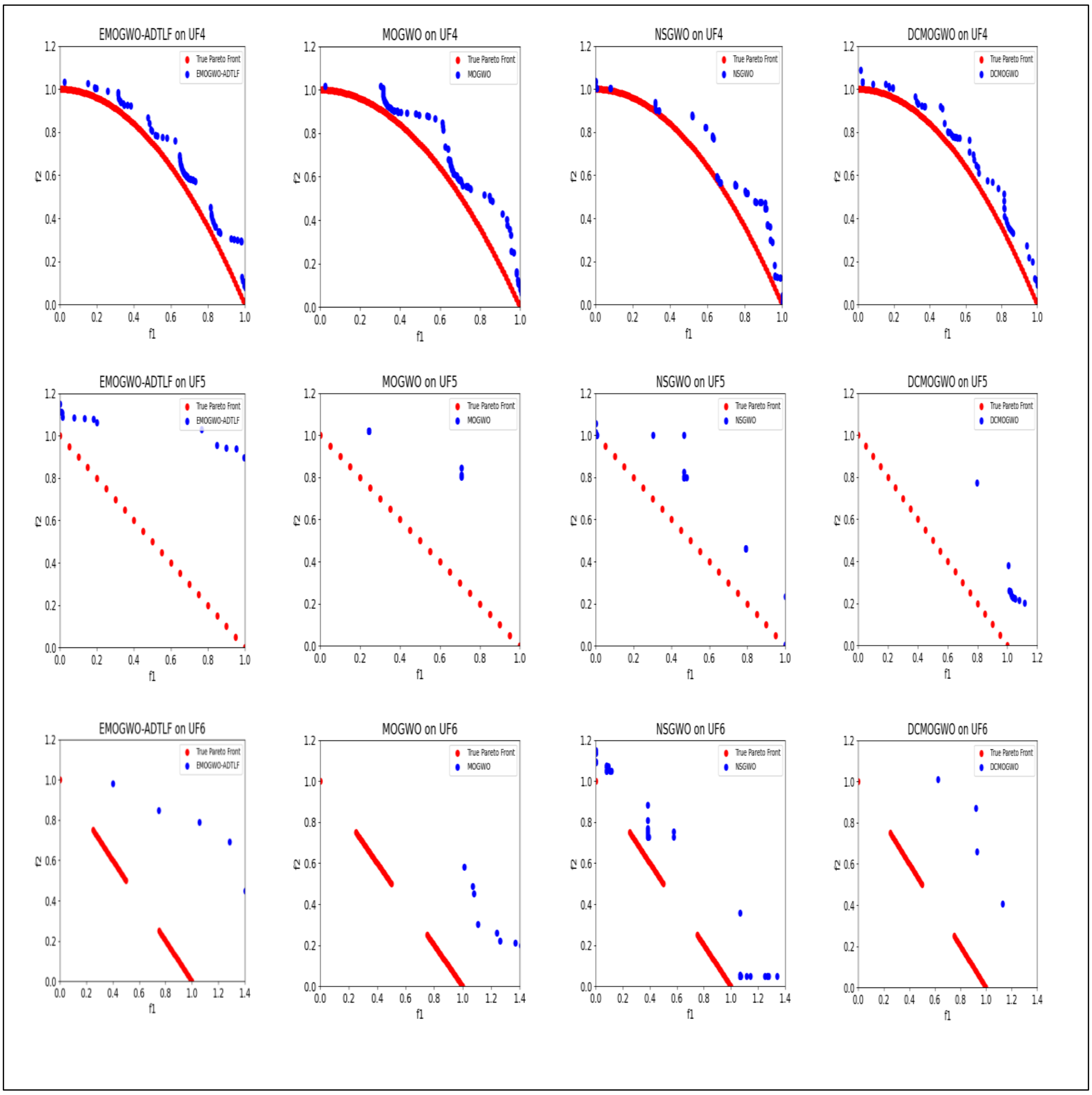

Fig. 3: Graph of test functions UF4, UF5 and UF6

The plots of Pareto solutions for the algorithms are shown in *fig. 4*. For the UF7, EMOGWO-ADTLF shows better convergence, spread and distribution than the rest of the algorithms. MOGWO shows good spread but poor distribution while NSGWO and DCMOGWO struggle with both spread and distribution. In the three-dimension UF8 function, EMOGWO-ADTLF and NSGWO have better spread and distribution but not all the obtained Pareto fronts converge to the Pareto front. MOGWO has good convergence but struggles with spread and coverage. DCMOGWO has poor convergence, spread and distribution.

Overall, EMOGWO-ADTLF shows a consistent close approximation of the Pareto fronts in most of the test functions. NSGWO, MOGWO and DCMOGWO perform variably across the test functions. It shows reasonable approximations but sometimes suffers from spread and distribution. DCMOGWO and MOGWO’s performance reduces as the complexity of the functions increases.

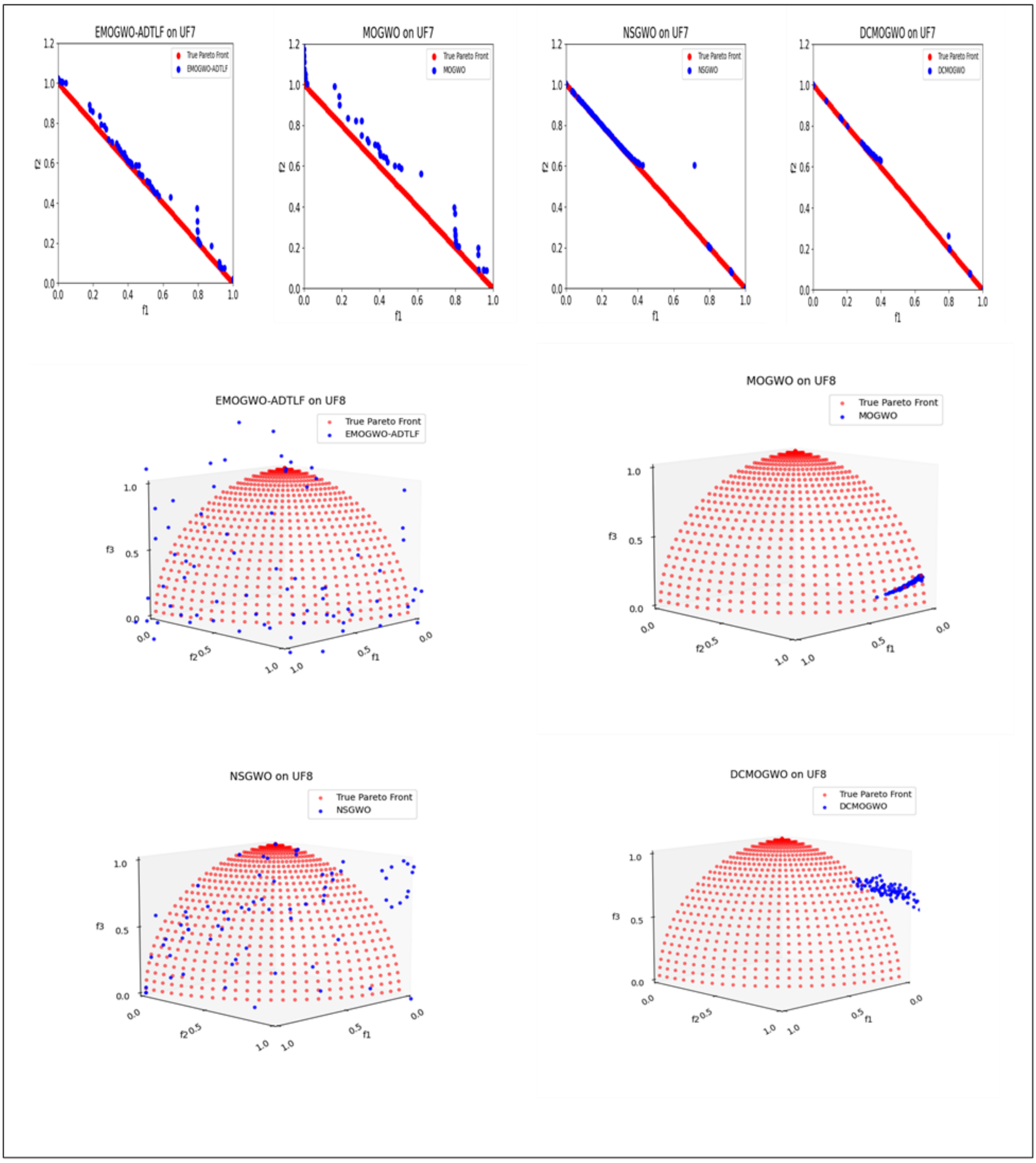

Fig. 4: Graph of test functions UF7 and UF8

4.1.1. Analysis of IGD and HV Values for EMOGWO-ADTLF and MOGWO Variants

The results of IGD values for EMOGWO-ADTLF, MOGWO, NSGWO and DCMOGWO are presented in Table 3. The statistical measures are the average (AVG), median (MDN), standard deviation (SD), best score (BS) and worst score (WS). In UF1, EMOGWO-ADTLF dominates in the performance metrics, obtaining the best values in average, median, best and worst score values. For this function, EMOGWO-ADTLF shows high diversity and convergence. This implies the algorithm can balance exploration and exploitation for this test function. DCMOGWO outperforms NSGWO and MOGWO in terms of average IGD but below EMOGWO-ADTLF. NSGWO has the best standard deviation value. For the UF2 function, EMOGWO-ADTLF has the best possible value, but its best average value is slightly below NSGWO. In UF3, UF4 and UF6 functions, EMOGWO-ADTLF dominates in both average and best scores. This again shows the strength of EMOGWO-ADTLF to provide convergence and diversity in these functions. NSGWO shows strength in closely approximating the Pareto fronts in the UF6 function. It dominates in four of the performance metrics. In UF7, EMOGWO-ADTLF shows its highest strength, dominating in all the performance metrics. It is an indication of the ability of MOGWO-ADTLF to handle complex problems. NSGWO has the lowest average, median, standard deviation, and worst values in UF8. This performance of EMOGWO-ADTLF closely follows NSGWO. Both MOGWO and DCMOGWO struggle to handle this complex problem. For this three-dimensional problem function, NSGWO and EMOGWO-ADTLF have better diversity and convergence.

Across all the functions, EMOGWO-ADTLF frequently performs better than other algorithms in terms of IGD, showing high effectiveness and efficiency. This is proof of convergence and diversity. It demonstrates the ability of EMOGWO-ADTLF to provide a balance between exploration and exploitation as well as closely approximating the Pareto fronts. NSGWO also shows competitive performance especially in complex problems.

Table 4 presents the HV value for EMOGWO-ADTLF, MOGWO, NSGWO and DCMOGWO. EMOGWO-ADTLF has the highest values in terms of average, median and worst values in UF1 and UF2. It also has the highest best value in UF1. This shows that the Pareto solutions of EMOGWO-ADTLF cover the objective space effectively in both functions. MOGWO and DCMOGWO are the second-best performers for UF1 with NSGWO providing competitive performance to EMOGWO-ADTLF in UF2, obtaining the highest best value. The dominance of EMOGWO-ADTLF continues in both UF3 AND UF4, obtaining the highest scores in average, median and best values. DCMOGWO outperforms MOGWO and NSGWO in the two test problems. NSGWO also covers the objective space effectively in UF5 and UF6. It obtains the highest HV values in four of the performance metrics in both functions. In UF7 and UF8, EMOGWO-ADTLF again shows dominant performance in the performance metrics. It also indicates the strength of EMOGWO-ADTLF in handling complex problems more effectively. The performance of EMOGWO-ADTLF in these test functions is followed by NSGWO.MOGWO and DCMOGWO are the worst performing algorithms in UF7 and UF8 respectively.

EMOGWO-ADTLF more often performs better than other algorithms in terms of HV, obtaining the highest or near-highest HV values. It also gives competitive standard deviation values. This shows how effective EMOGWO-ADTLF covers the objective space and is a good algorithm for different optimization problems. NSGWO also performs well in certain functions, but its performance fluctuates depending on the problem being considered.

Table 3. Analysis of IGD Values

FUNCT-ION	STATI-STICS	EMOGWO-ADTLF	MOGWO	NSGWO	DCMOGWO
UF1	AVG	**5.7969E-02**	7.1311E-02	7.9473E-02	6.7857E-02
	MDN	**5.7273E-02**	8.4320E-02	7.6504E-02	6.5035E-02
	SD	1.4157E-02	2.0105E-02	**4.7193E-03**	1.4144E-02
	BS	**4.0989E-02**	4.2911E-02	7.5781E-02	5.2119E-02
	WS	**7.5646E-02**	8.6701E-02	8.6134E-02	8.6418E-02
UF2	AVG	3.3437E-02	4.5888E-02	**3.2598E-02**	4.1171E-02
	MDN	**3.2656E-02**	4.5811E-02	3.2813E-02	4.1180E-02
	SD	1.7721E-03	2.3011E-03	3.1591E-04	**2.9645E-04**
	BS	**3.1764E-02**	4.3109E-02	3.2151E-02	4.0804E-02
	WS	3.5889E-02	4.8744E-02	**3.2829E-02**	4.1530E-02
UF3	AVG	**2.8641E-01**	3.4817E-01	3.8643E-01	3.0021E-01
	MDN	**3.2332E-01**	3.4253E-01	3.8653E-01	**3.2333E-01**
	SD	5.9279E-02	**1.1552E-02**	6.7217E-04	3.3181E-02
	BS	**2.0276E-01**	3.3771E-01	3.8556E-01	2.5329E-01
	WS	3.3313E-01	3.6427E-01	3.8720E-01	**3.2402E-01**
UF4	AVG	**4.9430E-02**	5.8564E-02	6.9335E-02	5.1609E-02
	MDN	**4.9253E-02**	5.3530E-02	6.5971E-02	5.1530E-02
	SD	1.1361E-03	1.0918E-02	7.1741E-03	**9.7808E-04**
	BS	**4.8136E-02**	4.8441E-02	6.2727E-02	5.0453E-02
	WS	**5.0902E-02**	7.3722E-02	7.9306E-02	5.2845E-02
UF5	AVG	**4.7367E-01**	4.7994E-01	1.7046E+00	5.4062E-01
	MDN	5.3407E-01	5.2411E-01	1.7113E+00	**5.2327E-01**
	SD	1.4897E-01	1.1658E-01	**1.0118E-02**	9.8080E-02
	BS	**2.6869E-01**	3.2030E-01	1.6903E+00	4.3011E-01
	WS	6.1826E-01	**5.9541E-01**	1.7122E+00	6.6847E-01
UF6	AVG	6.5347E-01	5.7803E-01	**2.0821E-01**	4.4295E-01
	MDN	6.5014E-01	5.8364E-01	**1.7291E-01**	4.3031E-01
	SD	**4.1635E-02**	7.9377E-02	5.7970E-02	1.0207E-01
	BS	6.0423E-01	4.7813E-01	**1.6178E-01**	3.2474E-01
	WS	7.0605E-01	6.7232E-01	**2.8994E-01**	5.7380E-01
UF7	AVG	**4.2915E-02**	1.4181E-01	5.7573E-02	6.1463E-02
	MDN	**4.7423E-02**	6.8108E-02	5.7461E-02	6.3971E-02
	SD	**9.0520E-03**	1.1060E-01	1.0112E-02	7.8416E-03
	BS	**3.0284E-02**	5.9182E-02	4.5245E-02	5.0854E-02
	WS	**5.1036E-02**	2.9813E-01	7.0013E-02	6.9564E-02
UF8	AVG	1.6637E-01	9.1158E-01	**1.5490E-01**	1.3326E+00
	MDN	1.6880E-01	9.1806E-01	**1.5476E-01**	9.1289E-01
	SD	1.1489E-02	2.6171E-02	**1.3674E-03**	6.4448E-01
	BS	**1.5124E-01**	8.7678E-01	1.5329E-01	8.4183E-01
	WS	1.7906E-01	9.3990E-01	**1.5663E-01**	2.2431E+00

Table 4. Analysis of HV Values

FUNCT-ION	STATI-STICS	EMOGWO-ADTLF	MOGWO	NSGWO	**DCMOGWO**
UF1	AVG	**1.0012E+00**	9.6216E-01	9.6603E-01	9.7232E-01
	MDN	**1.0095E+00**	9.4560E-01	9.6965E-01	9.7123E-01
	SD	2.5703E-02	2.9627E-02	**5.8174E-03**	1.8139E-02
	BS	**1.0278E+00**	1.0038E+00	9.7063E-01	9.9507E-01
	WS	**9.6647E-01**	9.3711E-01	9.5783E-01	9.5068E-01
UF2	AVG	**1.0539E+00**	1.0317E+00	1.0516E+00	5.9579E-01
	MDN	**1.0550E+00**	1.0286E+00	1.0543E+00	5.9458E-01
	SD	1.9074E-03	6.0224E-03	4.9418E-03	**1.7843E-03**
	BS	1.0554E+00	1.0402E+00	**1.0558E+00**	5.9831E-01
	WS	**1.0512E+00**	1.0264E+00	1.0446E+00	5.9448E-01
UF3	AVG	**6.3035E-01**	5.1864E-01	4.4334E-01	6.1075E-01
	MDN	**5.6133E-01**	5.2197E-01	4.4324E-01	5.6512E-01
	SD	1.0299E-01	9.8704E-03	**4.8824E-04**	6.6666E-02
	BS	**7.7593E-01**	5.2871E-01	4.4398E-01	7.0501E-01
	WS	5.5379E-01	5.0523E-01	4.4279E-01	**5.6211E-01**
UF4	AVG	**6.7915E-01**	6.6562E-01	6.6183E-01	6.7300E-01
	MDN	**6.7905E-01**	6.7018E-01	6.6571E-01	6.7250E-01
	SD	2.2381E-03	1.3642E-02	7.6380E-03	**1.2247E-03**
	BS	**6.8195E-01**	6.7958E-01	6.6862E-01	6.7400E-01
	WS	**6.7647E-01**	6.4711E-01	6.5116E-01	6.7100E-01
UF5	AVG	7.6565E-01	8.8663E-01	**1.4245E+00**	6.7332E-01
	MDN	6.3858E-01	8.7735E-01	**1.4244E+00**	7.7173E-01
	SD	3.1557E-01	**1.2209E-01**	1.4060E-01	1.5154E-01
	BS	1.1997E+00	1.0406E+00	**1.5968E+00**	7.8899E-01
	WS	4.5868E-01	7.4196E-01	**1.2524E+00**	4.5925E-01
UF6	AVG	5.0265E-01	5.6163E-01	**1.2720E+00**	7.9419E-01
	MDN	5.1714E-01	5.1641E-01	**1.3061E+00**	7.4149E-01
	SD	**2.4046E-02**	6.8006E-02	6.2818E-02	1.1729E-01
	BS	5.2205E-01	6.5775E-01	**1.3259E+00**	9.5675E-01
	WS	4.6876E-01	5.1073E-01	**1.1839E+00**	6.8433E-01
UF7	AVG	**8.6040E-01**	7.3017E-01	8.4892E-01	8.2923E-01
	MDN	**8.5256E-01**	8.1323E-01	8.4898E-01	8.2506E-01
	SD	1.6666E-02	1.2832E-01	2.4668E-02	**1.1276E-02**
	BS	**8.8357E-01**	8.2836E-01	8.7910E-01	8.4465E-01
	WS	**8.4506E-01**	5.4891E-01	8.1867E-01	8.1799E-01
UF8	AVG	**2.4020E+00**	1.0239E+00	2.2615E+00	4.4310E-01
	MDN	**2.4011E+00**	1.0425E+00	2.2783E+00	2.8703E-01
	SD	**3.2163E-02**	5.6601E-02	4.5080E-02	4.3959E-01
	BS	**2.4419E+00**	1.0820E+00	2.3063E+00	1.0423E+00
	WS	**2.3631E+00**	9.4713E-01	2.1998E+00	0.0000E+00

4.2. Analysis of EMOGWO-ADTLF against Other Multi-Objective Optimizers

Table 5 presents the results of IGD values. EMOGWO-ADTLF dominates other algorithms in UF1 and UF2 functions. It achieves the lowest IGD value in terms of average, median, best and worst values. It also gives the lowest standard deviation value for UF2. In UF3, MODA outperforms all the algorithms, obtaining the best value in terms of average, median, standard deviation and worst scores. MOALO and EMOGWO-ADLLF provide competitive scores with MMA trailing all algorithms. EMOGWO-ADTLF obtains the lowest values in all the statistical metrics in UF4, showing more diversity and convergence in this problem. For the UF5, EMOGWO-ADTLF has the best values in overall best and average values. MOALO provides competitive performance, achieving the lowest median and worst value. MODA has the lowest standard deviation for this function. MOALO is the most dominant algorithm in the UF6 function. It has the best values for four of the metrics, including average values. EMOGWO-ADTLF again has the lowest values for all the performance metrics in UF7 and UF8. This is a clear indication of the ability of EMOGWO-ADTLF to approximate the Pareto fronts and provide a balance between exploitation and exploration in complex problems.

In all the IGD values, EMOGWO-ADTLF emerges as the top performer in most of the functions. It highlights the algorithm's effectiveness, efficiency, diversity and adaptability in different problems. The other algorithms also show competitiveness in a few test problems. MODA's overall performance is better than MOALO and MMA.

The analysis of HV values is shown in Table 6. In the UF1 and UF2 functions, EMOGWO-ADTLF has the highest values in average, median, best and worst values. It also has the best standard deviation value. This proves that EMOGWO-ADTLF has the best coverage and spread for these functions. MODA is the second-best performer for UF1 function with MOALO providing competitive performance to EMOGWO-ADTLF in UF2. MODA has dominant performance in the UF3 function, showing its ability to cover the objective space more effectively. The performance of MODA in UF3 is followed by MOALO and EMOGWO-ADTLF. EMOGWO-ADTLF again outperforms all the other algorithms in UF4. The second-best performer for this test function is MODA. In UF5 and UF6, MOALO is the top performer in terms of HV values. It has the best average, median and worst values. Its overall highest average value suggests that it effectively occupies the objective space. The performance of MOALO in test function UF5 is closely followed by EMOGWO-ADTLF. EMOGWO-ADTLF has the highest values in all the statistical metrics for test functions UF7 and UF8. This is an indication of the algorithm's ability to produce solutions that cover the objective space. The performance is also a proof of the algorithm to perform in complex and high-dimension problems. MODA's performance in these two test functions is better than MMA and MOALO.

In the HV analysis, EMOGWO-ADTLF obtained the best HV value in most of the test functions. It indicates the effectiveness of EMOGWO-ADTLF. It can produce diverse and quality solutions, with effective distribution in the objective space.

Table 5. Analysis of IGD Values

FUNCT-ION	STATI-STICS	EMOGWO-ADTLF	MMA	MOALO	MODA
UF1	AVG	**5.7969E-02**	5.0362E-01	1.2529E-01	8.0228E-02
	MDN	**5.7273E-02**	3.6851E-01	1.2531E-01	7.6724E-02
	SD	1.4157E-02	2.0709E-01	**5.0827E-03**	5.6834E-03
	BS	**4.0989E-02**	3.4614E-01	1.1906E-01	7.5715E-02
	WS	**7.5646E-02**	7.9621E-01	1.3151E-01	8.8244E-02
UF2	AVG	**3.3437E-02**	4.6596E-01	1.0055E-01	1.3648E-01
	MDN	**3.2656E-02**	4.8933E-01	9.5645E-02	1.5505E-01
	SD	**1.7721E-03**	4.6445E-02	8.2915E-03	5.5666E-02
	BS	**3.1764E-02**	4.0111E-01	9.3785E-02	6.0944E-02
	WS	**3.5889E-02**	5.0743E-01	1.1223E-01	1.9345E-01
UF3	AVG	2.8641E-01	4.9969E-01	2.2076E-01	**1.6356E-01**
	MDN	3.2332E-01	3.6962E-01	2.5563E-01	**1.5164E-01**
	SD	5.9279E-02	2.0679E-01	7.3376E-02	**2.3194E-02**
	BS	2.0276E-01	3.3789E-01	**1.1869E-01**	1.4306E-01
	WS	3.3313E-01	7.9157E-01	2.8797E-01	**1.9599E-01**
UF4	AVG	**4.9430E-02**	4.0835E-01	1.2166E-01	9.0255E-02
	MDN	**4.9253E-02**	4.3332E-01	1.2421E-01	8.5741E-02
	SD	**1.1361E-03**	1.4044E-01	1.3229E-02	9.3001E-03
	BS	**4.8136E-02**	2.2522E-01	1.0433E-01	8.1813E-02
	WS	**5.0902E-02**	5.6650E-01	1.3643E-01	1.0321E-01
UF5	AVG	**4.7367E-01**	6.7863E-01	4.7680E-01	7.0163E-01
	MDN	5.3407E-01	7.8823E-01	**4.4111E-01**	7.1891E-01
	SD	1.4897E-01	1.9160E-01	1.0164E-01	**5.1014E-02**
	BS	**2.6869E-01**	4.0923E-01	3.7405E-01	6.3233E-01
	WS	6.1826E-01	8.3844E-01	**6.1523E-01**	7.5365E-01
UF6	AVG	6.5347E-01	7.6945E-01	**3.7119E-01**	4.7254E-01
	MDN	6.5014E-01	8.5334E-01	**3.5630E-01**	4.3815E-01
	SD	**4.1635E-02**	2.2029E-01	6.3267E-02	4.9282E-02
	BS	6.0423E-01	4.6767E-01	**3.0223E-01**	4.3723E-01
	WS	7.0605E-01	9.8734E-01	**4.5504E-01**	5.4223E-01
UF7	AVG	**4.2915E-02**	4.5262E-01	1.8774E-01	6.8319E-02
	MDN	**4.7423E-02**	4.6748E-01	1.8716E-01	6.3320E-02
	SD	**9.0520E-03**	3.2862E-02	2.1535E-02	1.1269E-02
	BS	**3.0284E-02**	4.0706E-01	1.6166E-01	5.7714E-02
	WS	**5.1036E-02**	4.8333E-01	2.1440E-01	8.3923E-02
UF8	AVG	**1.6637E-01**	6.1608E-01	5.9863E-01	2.8547E-01
	MDN	**1.6880E-01**	6.0978E-01	6.1293E-01	3.1074E-01
	SD	**1.1489E-02**	1.0405E-02	3.6773E-02	3.7108E-02
	BS	**1.5124E-01**	6.0772E-01	5.4818E-01	2.3300E-01
	WS	**1.7906E-01**	6.3075E-01	6.3479E-01	3.1266E-01

Table 6. Analysis of HV Analysis

FUNCT-ION	STATI-STICS	EMOGWO-ADTFL	MMA	MOALO	MODA
UF1	AVG	**1.0012E+00**	5.0912E-01	8.9127E-01	9.5553E-01
	MDN	**1.0095E+00**	5.5278E-01	9.0266E-01	9.6276E-01
	SD	2.5703E-02	1.5508E-01	2.1287E-02	**1.0963E-02**
	BS	**1.0278E+00**	6.7343E-01	9.0971E-01	9.6380E-01
	WS	**9.6647E-01**	3.0116E-01	8.6144E-01	9.4004E-01
UF2	AVG	**1.0539E+00**	6.3847E-01	9.3339E-01	8.8999E-01
	MDN	**1.0550E+00**	6.3588E-01	9.4018E-01	8.7718E-01
	SD	**1.9074E-03**	2.2371E-02	1.5247E-02	8.5429E-02
	BS	**1.0554E+00**	6.6707E-01	9.4771E-01	1.0004E+00
	WS	**1.0512E+00**	6.1246E-01	9.1227E-01	7.9236E-01
UF3	AVG	6.3035E-01	5.1276E-01	7.5304E-01	**8.3905E-01**
	MDN	5.6133E-01	5.5765E-01	6.8235E-01	**8.6249E-01**
	SD	1.0299E-01	1.5291E-01	1.1236E-01	**5.2533E-02**
	BS	7.7593E-01	6.7351E-01	**9.1163E-01**	8.8839E-01
	WS	5.5379E-01	3.0711E-01	6.6512E-01	**7.6628E-01**
UF4	AVG	**6.7915E-01**	2.4300E-01	5.3547E-01	6.0787E-01
	MDN	**6.7905E-01**	1.9718E-01	5.2094E-01	6.1510E-01
	SD	**2.2381E-03**	7.5948E-02	2.6513E-02	1.8308E-02
	BS	**6.8195E-01**	3.5004E-01	5.7267E-01	6.2578E-01
	WS	**6.7647E-01**	1.8178E-01	5.1280E-01	5.8272E-01
UF5	AVG	7.6565E-01	4.3639E-01	**8.0392E-01**	3.4721E-01
	MDN	6.3858E-01	2.7468E-01	**9.0798E-01**	3.5879E-01
	SD	3.1557E-01	2.6200E-01	1.4734E-01	**7.9437E-02**
	BS	**1.1997E+00**	8.0596E-01	9.0822E-01	4.3819E-01
	WS	4.5868E-01	2.2854E-01	**5.9556E-01**	2.4464E-01
UF6	AVG	5.0265E-01	3.9110E-01	**8.4325E-01**	7.4843E-01
	MDN	5.1714E-01	1.4209E-01	**8.5421E-01**	8.4941E-01
	SD	**2.4046E-02**	3.9479E-01	8.2956E-02	1.4449E-01
	BS	5.2205E-01	**9.4837E-01**	9.3892E-01	8.5178E-01
	WS	4.6876E-01	8.2837E-02	**7.3661E-01**	5.4410E-01
UF7	AVG	**8.6040E-01**	4.2255E-01	6.5250E-01	7.9854E-01
	MDN	**8.5256E-01**	4.0710E-01	6.6287E-01	8.0661E-01
	SD	**1.6666E-02**	2.6991E-02	4.7455E-02	2.3511E-02
	BS	**8.8357E-01**	4.6050E-01	7.0474E-01	8.2244E-01
	WS	**8.4506E-01**	4.0005E-01	5.8990E-01	7.6657E-01
UF8	AVG	**2.4020E+00**	1.0252E+00	1.0918E+00	1.6236E+00
	MDN	**2.4011E+00**	1.0421E+00	9.4338E-01	1.5231E+00
	SD	**3.2163E-02**	5.3716E-02	2.1962E-01	3.0756E-01
	BS	**2.4419E+00**	1.0809E+00	1.4023E+00	2.0404E+00
	WS	**2.3631E+00**	9.5263E-01	9.2974E-01	1.3074E+00

4.3. Summary of IGD and HV Values for Test Function UF1 to UF8

In this section, average IGDs and HVs are used to rank all algorithms. The use of average values for ranking provides the overall performance of the algorithms. It is the statistical measure that gives a clear indication of an algorithm's convergence towards the Pareto solutions. It also provides details about the efficiency, spread coverage and diversity of an algorithm. Tables 7 and 8 provide the average IGD and HV values of all the algorithms used in this study.

EMOGWO-ADTLF shows consistently high performance in IGD and HV values. It ranks first in both performance metrics with overall rank scores of 17 and 18 in terms of IGD and HV values respectively. For IGD values, NSGWO and DCMOGWO placed second and third positions with overall rank scores of 26 and 28. MMA is the worst performing algorithm. NSGWO and MOGWO are the second and third best algorithms for the HV values ranking. MMA again is the worst performing algorithm in the HV score ranking with MOALO and DCMOGWO placing fourth.

In general, EMOGWO-ADTLF dominates both IGD and HV values. This shows that EMOGWO-ADLF has the best diversity, convergence and coverage among all the algorithms. The best diversity proves the ability of the algorithm to balance between exploitation and exploration in most of the functions. The variability of the other algorithms across the test function shows that their performances depend on the problem being analysed. The consistently low performance of MMA is a sign that it needs to be improved to be able to correctly approximate the Pareto fronts.

Table 7. Ranking of IGD Values

FUNCT-ION	EMOGWO-ADTLF	MOGWO	NSGWO	DCMOG-WO	MMA	MOALO	MODA
UF1	1	3	4	2	7	6	5
UF2	2	4	1	3	7	5	6
UF3	3	5	6	4	7	2	1
UF4	1	3	4	2	7	6	5
UF5	1	3	7	4	5	2	6
UF6	6	5	1	3	7	2	4
UF7	1	5	2	3	7	6	4
UF8	2	6	1	7	5	4	3
TOTAL	17	34	26	28	52	33	34
TOTAL RANK	**1**	**5**	**2**	**3**	**7**	**4**	**5**

Table 8. Ranking of HV Values

FUNCT-ION	EMOGWO-ADTFL	MOGWO	NSGWO	DCMOG-WO	MMA	MOALO	MODA
UF1	1	2	4	2	7	6	5
UF2	1	3	2	7	6	4	5
UF3	3	5	7	4	6	2	1
UF4	1	3	4	2	7	6	5
UF5	4	2	1	5	6	3	7
UF6	6	5	1	3	7	2	4
UF7	1	5	2	3	7	6	4
UF8	1	6	2	7	5	4	3
TOTAL	18	31	23	33	51	33	34
TOTAL RANK	**1**	**3**	**2**	**4**	**7**	**4**	**6**

4.4. Wilcoxon Signed-Rank Test on Average IGD Values

The Wilcoxon signed-rank test was employed to assess whether the optimization performance of EMOGWO-ADTLF is statistically different from other algorithms. The test was conducted at a significance level of 0.05. The data used for the test were obtained from Tables 3 and 5, and the outcomes are summarized in Table 7. In this table, $R+$ represents the sum of ranks for positive differences, and $R-$ represents the sum of ranks for negative differences. The n/w/l/t column provides the following information: n is the total number of test functions considered, w is the number of functions where EMOGWO-ADTLF outperformed the compared algorithm, t is the number of functions where both algorithms exhibited equivalent performance, and l is the number of functions where EMOGWO-ADTLF underperformed the compared algorithm.

The results of the test are presented in Table 9. The results reveal that the p-value for the comparison between EMOGWO-ADTLF and MMA is 0.00781, below the significance level of 0.05. This indicates that EMOGWO-ADTLF exhibits a statistically significant performance improvement compared to MMA across the test functions. On the other hand, the p-values for the comparisons with MOGWO, NSGWO, DCMOGWO, MOALO, and MODA are above the significance level of 0.05, suggesting that the differences in performance between EMOGWO-ADTLF and these algorithms are not statistically significant. However, an examination of the n/w/l/t column reveals that EMOGWO-ADTLF outperformed MOGWO in 7 out of 8 functions, NSGWO in 5 out of 8 functions, DCMOGWO in 7 out of 8 functions, MOALO in 6 out of 8 functions, and MODA in 6 out of 8 functions. These results demonstrate that while the differences may not be statistically significant, EMOGWO-ADTLF exhibits superior optimization performance compared to these algorithms in most of the test functions.

Table 9. Wilcoxon Signed-Ranked Test of IGD Values

Algorithms	R+	R-	p-value	n/w/l/t	Significant?
EMOGWO-ADTLF vs MOGWO	30	6	0.1094	8/7/1/0	No
EMOGWO-ADTLF vs NSGWO	26	10	0.3125	8/5/3/0	No
EMOGWO-ADTLF vs DCMOGWO	29	7	0.1484	8/7/1/0	No
EMOGWO-ADTLF vs MMA	36	0	0.00781	8/8/0/0	Yes
EMOGWO-ADTLF vs MOALO	29	7	0.25	8/6/2/0	No
EMOGWO-ADTLF vs MODA	23	13	0.5469	8/6/2/0	No

4.5. Analysis of Algorithms Using Real-World Engineering Problems

The test for diversity, convergence, and coverage of EMOGWO-ADTLF is determined in this section. The IGD and HV values are compared with three well known algorithms in Engineering applications namely MMA, MOALO and MODA. For each engineering problem, the Pareto front is determined by combining the obtained solutions from all algorithms into one dataset and performing non-dominated sorting, using Python package DEAP. Tables 10 and 11 present the IGD and HV analysis for the Welded Beam and Disc Brake Engineering Designs respectively.

For the IGD analysis, EMOGWO-ADLLF dominates in both design problems, obtaining the lowest overall average values. EMOGWO-ADTLF also obtains the best values in three statistical metrics for the HV analysis for both problems. EMOGWO-ADTLF shows high diversity, coverage and convergence for these engineering problems. The results suggest that EMOGWO-ADTLF can maintain a balance between exploration and exploitation in real-world constrained Engineering problems. MMA shows an improvement in its performance in the unconstrained test functions. This indicates that MMA can be useful in engineering applications.

Table 10. IGD Values of Engineering Problems

FUNCT-ION	STATI-STICS	EMOGWO-ADTLF	MMA	MODA	MOALO
Welded Beam Design	AVG	**3.7400E-03**	4.1674E-03	4.0994E-03	3.8535E-03
	MDN	**3.7936E-03**	4.3654E-03	4.2613E-03	3.8507E-03
	SD	1.3111E-04	3.2143E-04	4.4301E-04	7.8066E-04
	BS	**3.5594E-03**	3.7140E-03	3.4943E-03	**2.8988E-03**
	WS	**3.8669E-03**	4.4227E-03	4.5426E-03	4.8110E-03

FUNCT-ION	STATI-STICS	EMOGWO-ADTLF	MMA	MODA	MOALO
Disc Brake Design	AVG	**1.8066E-03**	1.8816E-03	2.0188E-03	1.8899E-03
	MDN	**1.7089E-03**	1.9505E-03	1.9902E-03	1.7166E-03
	SD	1.4200E-04	**1.0507E-04**	1.6469E-04	1.1086E-04
	BS	**1.7034E-03**	1.7331E-03	1.8329E-03	1.7087E-03
	WS	2.0074E-03	**1.9611E-03**	2.1032E-03	2.2445E-03

Table 11. HV Values of Engineering Problems

FUNCT-ION	STATI-STICS	EMOGWO-ADTFL	MMA	MODA	MOALO
Welded Beam Design	AVG	**5.7618E-01**	5.6272E-01	4.6346E-01	5.7011E-01
	MDN	**5.7012E-01**	5.6783E-01	4.6209E-01	5.6510E-01
	SD	4.5864E-02	8.5084E-03	**7.7780E-03**	7.8335E-03
	BS	**6.3514E-01**	5.6960E-01	4.7360E-01	5.8117E-01
	WS	5.2329E-01	5.5073E-01	4.5469E-01	**5.6405E-01**
Disc Brake Design	AVG	**4.1770E+01**	4.1309E+00	3.8351E+01	3.0851E+01
	MDN	**4.1531E+01**	4.1305E+00	3.8949E+01	3.0879E+01
	SD	7.5412E-01	**1.6374E-02**	9.1732E-01	8.2573E+00
	BS	**4.2790E+01**	4.1512E+00	3.9049E+01	4.0950E+01
	WS	4.0990E+01	**4.1111E+00**	3.7055E+01	2.0724E+01

5. CONCLUSION

This study has developed an enhanced MOGWO using adaptive population parameter tuning and levy flight theories. It solves issues in multi-objective optimization including parameter tuning. The analysis of IGD and HV values of EMOGWO-ADTLF across different test functions and engineering design problems shows its dominance over existing nature-inspired algorithms. EMOGWO-ADTLF ranks first in both IGD and HV values when compared to MOGWO, NSGWO, DCMOGWO, MMA, MOALO and MODA. This demonstrates the ability of the proposed algorithm to correctly approximate the Pareto fronts and cover the objective space. This indicates that EMOGWO-ADTLF outperforms all other algorithms in terms of diversity, convergence, and coverage. Its superior diversity demonstrates the algorithm's capability to effectively balance exploration and exploitation.

The work shows the potency of adaptive diversity approaches and Levy flight theories in developing robust algorithms for complex real-world problems. It offers a robust tool for solving complex multi-objective problems with improved parameter tuning. Future Studies should concentrate on the scalability of the MOGWO algorithm.

REFERENCES

[1] Mirjalili S, Saremi S, Mirjalili SM, Coelho LDS. *Multi-objective grey wolf optimizer: A novel algorithm for multi-criterion optimization*. Expert Syst Appl 2016; 47:106–19. https://doi.org/10.1016/j.eswa.2015.10.039.

[2] Yang Y, Yang B, Wang S, Jin T, Li S. *An enhanced multi-objective grey wolf optimizer for service composition in cloud manufacturing*. Applied Soft Computing Journal 2020;87. https://doi.org/10.1016/j.asoc.2019.106003.

[3] Huy THB, Kim D, Vo DN. *Multiobjective Optimal Power Flow Using Multiobjective Search Group Algorithm.* IEEE Access 2022; 10:77837–56. https://doi.org/10.1109/ACCESS.2022.3193371.

[4] Avvari RK, Kumar D M V. *A Novel Hybrid Multi-Objective Evolutionary Algorithm for Optimal Power Flow in Wind, PV, and PEV Systems*. Journal of Operation and Automation in Power Engineering 2023; 11:130–43. https://doi.org/10.22098/JOAPE.2023.10371.1746.

[5] Pandya SB, Jiriwala RJ. *Non Dominated Sorting Dragonfly Algorithm For Multi-Objectives Optimal Power Flow*. 2019 Innovations in Power and Advanced Computing Technologies (i-PACT), 2019. https://doi.org/10.1109/i-PACT44901.2019.8960065.

[6] Kahraman HT, Akbel M, Duman S. *Optimization of Optimal Power Flow Problem Using Multi-Objective Manta Ray Foraging Optimizer*. Appl Soft Comput 2022;116. https://doi.org/10.1016/j.asoc.2021.108334.

[7] Vijaya Bhaskar K, Ramesh S, Karunanithi K, Raja SP. *Multi-Objective Optimal Power Flow Solutions Using Improved Multi-Objective Mayfly Algorithm (IMOMA).* Journal of Circuits, Systems and Computers 2023;32. https://doi.org/10.1142/S0218126623502006.

[8] Linnala M, Hämäläinen J. *Multiobjective optimisation in papermaking applications*. vol. 25. 2016.

[9] Pazhaniraja N, Basheer S, Thirugnanasambandam K, Ramalingam R, Rashid M, Kalaivani J. *Multi-objective Boolean grey wolf optimization based decomposition algorithm for high-frequency and high-utility itemset mining*. AIMS Mathematics 2023; 8:18111–40. https://doi.org/10.3934/math.2023920.

[10] Dilip L, Bhesdadiya R, Trivedi I, Jangir P. *Optimal Power Flow Problem Solution Using Multi-objective Grey Wolf Optimizer Algorithm*. Lecture Notes in Networks and Systems, vol. 19, Springer; 2018, p. 191–201. https://doi.org/10.1007/978-981-10-5523-2_18.

[11] Kamalova A, Navruzov S, Qian D, Lee SG. *Multi-robot exploration based on Multi-Objective Grey Wolf Optimizer.* Applied Sciences (Switzerland) 2019;9. https://doi.org/10.3390/app9142931.

[12] Wang C, Zhang S, Xiao L, Fu T. *Wind speed forecasting based on multi-objective grey wolf optimisation algorithm, weighted information criterion, and wind energy conversion system*: A case study in Eastern China. Energy Convers Manag 2021;243.

https://doi.org/10.1016/j.enconman.2021.114402.

[13] Zhu W, Guo J, Zhao G, Zeng B. *Optimal sizing of an island hybrid microgrid based on improved multi-objective grey wolf optimizer*. Processes 2020; 8:1–24. https://doi.org/10.3390/pr8121581.

[14] Makhadmeh SN, Al-Betar MA, Al-Obeidat F, Alomari OA, Abasi AK, Tubishat M, et al. *A multi-objective grey wolf optimizer for energy planning problem in smart home using renewable energy systems*. Sustainable Operations and Computers 2024; 5:88–101. https://doi.org/10.1016/j.susoc.2024.04.001.

[15] Nuaekaew K, Artrit P, Pholdee N, Bureerat S. *Optimal reactive power dispatch problem using a two-archive multi-objective grey wolf optimizer*. Expert Syst Appl 2017; 87:79–89. https://doi.org/10.1016/j.eswa.2017.06.009.

[16] Safari FM, Etebari F, Chobar AP. *Modeling and Optimization of a Tri-objective Transportation-Location-Routing Problem considering route reliability*: using MOGWO, MOPSO, MOWCA, and NSGA-II. Journal of Optimization in Industrial Engineering 2021; 14:99–114. https://doi.org/10.22094/JOIE.2020.1893849.1730.

[17] Tlili S, Mnasri S, Val T. *An improved multi-objective Grey Wolf Optimizer* 2022. https://doi.org/10.21203/rs.3.rs-2267221/v1.

[18] Dada EG, Joseph SB, Oyewola DO, Fadele AA, Chiroma H, Abdulhamid SM. *Application of Grey Wolf Optimization Algorithm: Recent Trends, Issues, and Possible Horizons*. Gazi University Journal of Science 2022; 35:485–504. https://doi.org/10.35378/gujs.820885.

[19] Zhang X qing, Ming Z feng. *An optimized grey wolf optimizer based on a mutation operator and eliminating-reconstructing mechanism and its application*. Frontiers of Information Technology and Electronic Engineering 2017; 18:1705–19. https://doi.org/10.1631/FITEE.1601555.

[20] Tian X, Hu Q, Liu CA. *An improved Multi-objective Grey Wolf Optimization Algorithm Based on Multiple Strategies.* Proceedings - 2022 International Conference on Computer Engineering and Artificial Intelligence, ICCEAI 2022, Institute of Electrical and Electronics Engineers Inc.; 2022, p. 74–8. https://doi.org/10.1109/ICCEAI55464.2022.00024.

[21] Gu W. *An improved multi-objective grey wolf optimization algorithm with dynamic chaos local search mechanism,* 2020, p. 2020–4. https://doi.org/10.1109/ITAIC49862.2020.9338760.

[22] Guo MW, Wang JS, Zhu LF, Guo SS, Xie W. *An Improved Grey Wolf Optimizer Based on Tracking and Seeking Modes to Solve Function Optimization Problems*. IEEE Access 2020; 8:69861–93. https://doi.org/10.1109/ACCESS.2020.2984321.

[23] Yang Z, Liu C, Qian W. *An improved multi-objective grey wolf optimization algorithm for fuzzy blocking flow shop scheduling problem*. 2017 IEEE 2nd Advanced Information Technology, Electronic and Automation Control Conference (IAEAC), IEEE; 2017, p. 661–7. https://doi.org/10.1109/IAEAC.2017.8054099.

[24] Al-Tashi Q, Abdulkadir SJ, Rais HM, Mirjalili S, Alhussian H, Ragab MG, et al. *Binary Multi-Objective Grey Wolf Optimizer for Feature Selection in Classification*. IEEE Access 2020; 8:106247–63. https://doi.org/10.1109/ACCESS.2020.3000040.

[25] Jangir P, Jangir N. *A new Non-Dominated Sorting Grey Wolf Optimizer (NS-GWO) algorithm: Development and application to solve engineering designs and economic constrained emission dispatch problem with integration of wind power*. Eng Appl Artif Intell 2018;72:449–67. https://doi.org/10.1016/j.engappai.2018.04.018.

[26] Fatima A, Javaid N, Butt AA, Sultana T, Hussain W, Bilal M, et al. *An enhanced multi-objective gray wolf optimization for virtualmachine placement in cloud data centers*. Electronics (Switzerland) 2019;8. https://doi.org/10.3390/electronics8020218.

[27] Jiang K, Ni H, Han R, Wang X. *An improved multi-objective grey wolf optimizer for dependent task scheduling in edge computing*. International Journal of Innovative Computing, Information and Control 2019;15:2289–304. https://doi.org/10.24507/ijicic.15.06.2289.

[28] Ahmadi B, Arias NB, Hoogsteen G, Hurink JL. *Multi-objective Advanced Grey Wolf optimization Framework for Smart Charging Scheduling of EVs in Distribution Grids*. 2022 57th International Universities Power Engineering Conference: Big Data and Smart Grids, UPEC 2022 - Proceedings, Institute of Electrical and Electronics Engineers Inc.; 2022. https://doi.org/10.1109/UPEC55022.2022.9917961.

[29] Yang Z, Liu C. *A hybrid multi-objective gray wolf optimization algorithm for a fuzzy blocking flow shop scheduling problem*. Advances in Mechanical Engineering 2018;10. https://doi.org/10.1177/1687814018765535.

[30] Mirjalili S, Mirjalili SM, Lewis A. *Grey Wolf Optimizer. Advances in Engineering Software* 2014; 69:46–61. https://doi.org/10.1016/j.advengsoft.2013.12.007.

[31] Sudholt D. *The benefits of population diversity in evolutionary algorithms: A survey of rigorous runtime analyses*. Natural Computing Series, Springer; 2020, p. 359–404. https://doi.org/10.1007/978-3-030-29414-4_8.

[32] Chawla M, Duhan M. *Levy Flights in Metaheuristics Optimization Algorithms–A Review*. Applied Artificial Intelligence 2018; 32:802–21. https://doi.org/10.1080/08839514.2018.1508807.

[33] Wu L, Wu J, Wang T. *Enhancing grasshopper optimization algorithm (GOA) with levy flight for engineering applications.* Sci Rep 2023;13. https://doi.org/10.1038/s41598-022-27144-4.

[34] Zhang Q, Zhou A, Zhao S, Suganthan PN, Liu W, Tiwari S. *Multiobjective optimization Test Instances for the CEC 2009 Special Session and Competition*. 2009.

[35] Yang XS. *Multiobjective firefly algorithm for continuous optimization*. Eng Comput 2013;29:175–84. https://doi.org/10.1007/s00366-012-0254-1.

[36] Mirjalili S, Jangir P, Saremi S. *Multi-objective ant lion optimizer: a multi-objective optimization algorithm for solving engineering problems*. Applied Intelligence 2017; 46:79–95. https://doi.org/10.1007/s10489-016-0825-8.

[37] Sierra MR, Coello CAC. *Improving PSO-Based Multi-objective Optimization Using Crowding, Mutation and -Dominance*. Springer-Verlag Berlin Heidelberg 2005; LNCS 3410:505–19.

[38] Zitzler E, Künzli S. *Indicator-Based Selection in Multiobjective Search*. 2004.

[39] Zervoudakis K, Tsafarakis S. *A mayfly optimization algorithm*. Comput Ind Eng 2020;145. https://doi.org/10.1016/j.cie.2020.106559.

[40] Mirjalili S. Dragonfly algorithm*: a new meta-heuristic optimization technique for solving single-objective, discrete, and multi-objective problems*. Neural Comput Appl 2016; 27:1053–73. https://doi.org/10.1007/s00521-015-1920-1.