

DYNAMICS MODELING AND CONTROL SYSTEM DEFINITION FOR VIBRATION ATTENUATION OF A THIN TITANIUM PLATE WITH MATLAB AND ANSYS

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Abstract: *Vibration artefacts that appear at small dimension components are frequently encountered in multiple domains such electronic equipment production, surgical and high precision medical applications, optical applications high quality photo/video equipment. The main objective of this work is to develop a reduced model that accurately represents the dynamics of a small dimension titanium plate and define a suitable vibration control system. With the aid of powerful engineering software instruments, the mathematical model of the studied object is defined with MATLAB after it was modeled and analyzed with ANSYS (mode shape analysis). The mathematical model is developed based on the eigenvectors and eigenvalues extracted from ANSYS and it is called the state space model which is then transformed into a transfer function for simplicity reasons. The mathematical model is then integrated in Simulink and the vibration velocity is evaluated, followed by the definition of a control system comprised of a piezoelectric actuator and PID controller. The evaluation of the system was performed by applying a step signal at the input of the plant and the velocity response is observed comparatively in three configurations as listed: small dimension plate response, plate and piezoelectric actuator and full plant (titanium plate, actuator and PID controller).*

1. INTRODUCTION

The vibration artefacts that appear at small dimension components are frequently encountered in multiple domains such electronic equipment production, surgical and high precision medical applications, optical applications high quality photo/video equipment. In general terms, the vibration that can be detected at different components is harmful to the

equipment and can lead to irremediable defects or they can decrease the performance of the equipment. Considering the previous mentions, the study of the vibration reduction technique is of a major importance and took the attention of numerous researchers becoming a subject of interest at a global level. The work of [1], [2], [3], [4] stood at the base of this paper as they are focusing on developing active vibration attenuation techniques with piezoelectric actuators.

The vibration reduction can be obtained by using two different techniques such as passive and active. The first mentioned technique can't be implemented in small scale applications due to different factors hence for reduced dimension applications the only applicable techniques are the ones which are using active components such as piezoelectric actuators. Beside the actuators which are operating based on the piezoelectric effect, in vibration control systems can also be used electro-mechanical, electrical, electrodynamic hydraulic and pneumatic actuators, but all the listed types are of large dimensions and are only suitable for large scale applications.

The main objective of this work is to develop an active vibration control system for small scale applications. Considering the fact that the modeling and analysis techniques do often induce errors in the results, for simplicity purposes in this paper it is presented a titanium thin plate with a length of 15 mm, width of 7 mm and a thickness of 0.5 mm.

The mathematical model of the titanium plate with the previously listed dimensions is obtained with the help of MATLAB software from the eigenvectors and eigenvalues extracted from ANSYS Mode Shape Analysis tool with the Block Lanczos method.

This paper is organized in six sections. The first part, the introduction with the objective and motivation description, the second section, with the physical description of the studied object and the third is presenting the mode shape analysis results. In the fourth part, the mathematical model extraction technique is described and in the fifth and sixth part, the MATLAB/SIMULINK project is presented, followed by the discussion of the results in the sixth part and conclusions in the end.

2. PHYSICAL MODEL DESCRIPTION

For this study, the research team has reached the conclusion that a small, simple object can be used for analysis. The physical characteristics of the thin titanium plate can be seen in *figure 1* and the mechanical properties in Table 1.

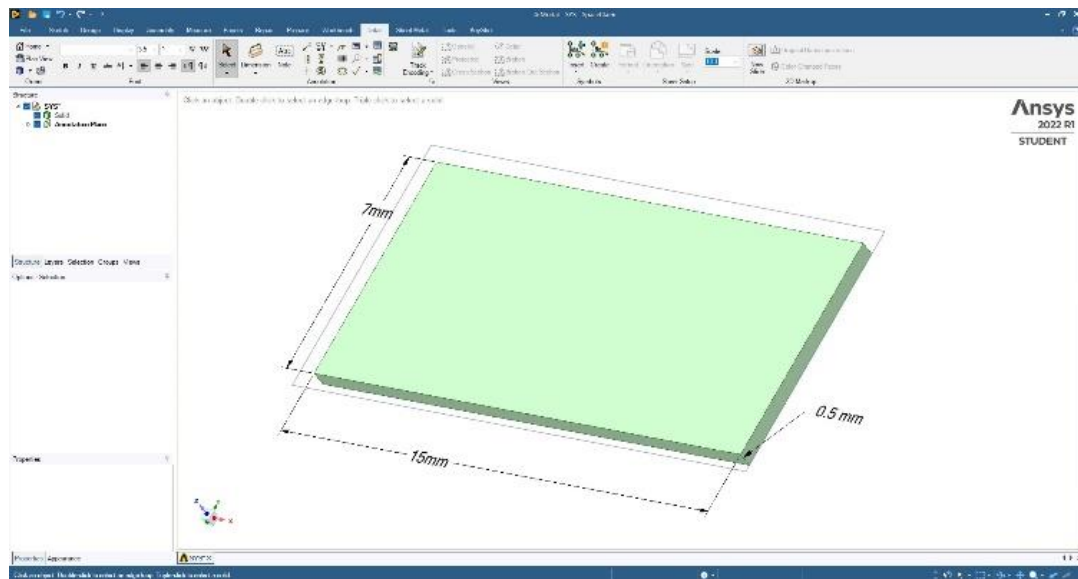


Fig. 1 Physical dimensions of the studied object

Table 1. Mechanical properties of the titanium alloy, selected as material of the thin plate.

Property	Value	Unit
Density	4620	Kg/m ³
Coefficient of thermal expansion	9.4E-06	°C ⁻¹
Young's Modulus	9.6E+10	Pa
Poisson's ratio	0.36	

3. MODE SHAPE ANALYSIS

As described in the introduction, an important step in finding the mathematical model of the studied object is to extract the eigenvalues and eigenvectors in order to be further processed. The eigenvalues and eigenvectors are extracted from ANSYS after the modal analysis is performed with specific instruments which are at the user's disposal in ANSYS. The eigenvalues and eigenvectors are extracted using the Block Lanczos method, as a standard technique. After extracting the necessary values by software specific means, the information corresponding to the X and Y displacement are isolated and only the Z component is used in the analysis. The natural frequencies for the first six modes of vibration were extracted and the corresponding eigenvalues for the Z displacement component were used. The natural frequencies (eigenvalues) are presented in table 2 but the eigenvectors can't be presented due to the high volume of extracted values. The studied component was split into 1340 nodes and 171 elements. For each natural frequency was extracted a number of 1340 values to form the eigenvector.

Table 2. Modes of vibration and natural frequencies.

Mode	1	2	3	4	5	6
Frequency [Hz]	1705.1	7383.5	10560	19965	23888	29425

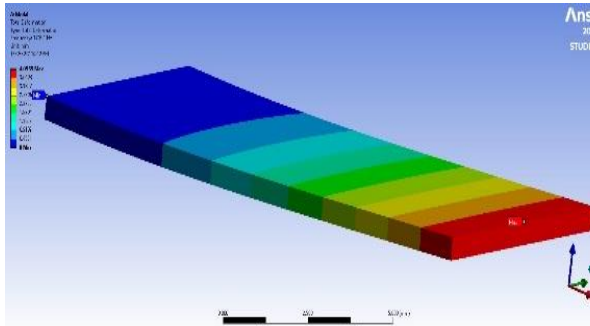


Fig. 2.1

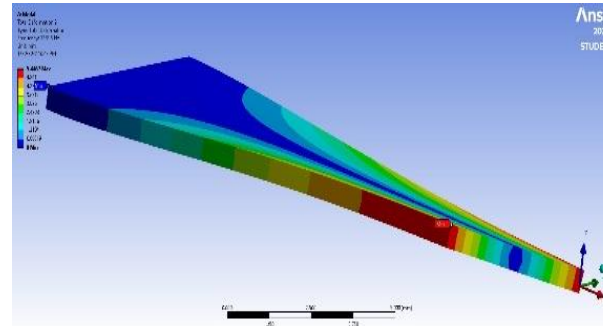


Fig. 2.2

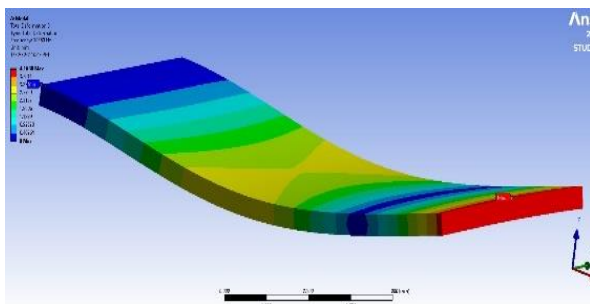


Fig. 2.3

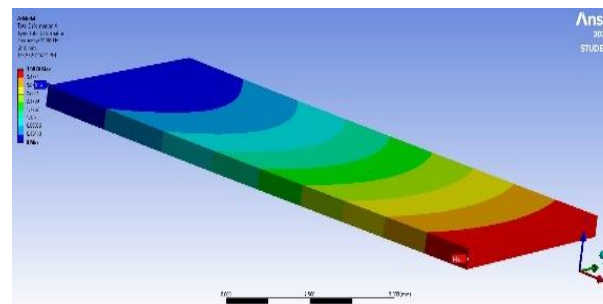


Fig. 2.4

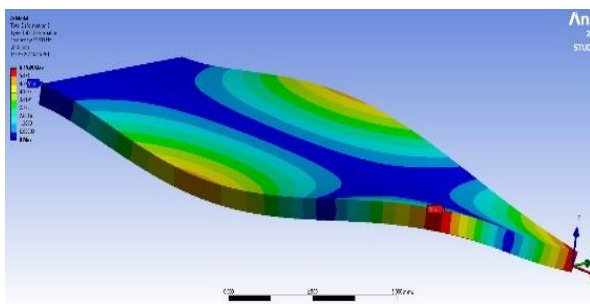


Fig. 2.5

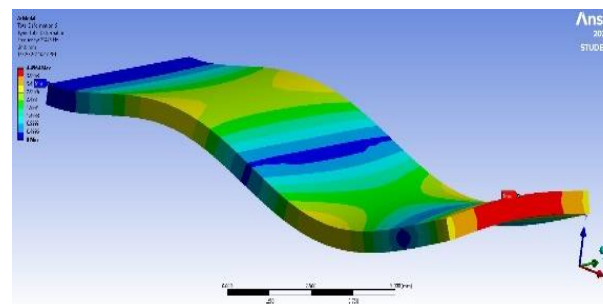


Fig. 2.6

Fig. 2.1 ... 2.6 Mode shape vibration pattern (all six modes). Index = number of the vibration mode

The general theory for the mode shape analysis is presented in [5]. The equation of motion of a multi DOF system under external forces is given by:

$$[m]\ddot{x} + [c]\dot{x} + [k]x = F \tag{1}$$

Considering that the mass, damping and stiffness matrices are non-diagonal, equation 1 is leading to n coupled second order differential equations. The eigenvalue problem for equation 1 is solved to uncouple the coupled equation set and the eigenvectors are obtained as $x(1), x(2) \dots x(n)$ which leads to the modal matrix of the system:

$$x_n = [x_{(1)} x_{(2)} \dots x_{(n)}] \quad (2)$$

For a system with multiple degree of freedom, taking as general assumption a proportional damping, the solution of the first equation can be expressed as:

$$x(t) = [x_n] \cdot x_p(t) \quad (3)$$

Having $x_p(t)$ the displacement in principal coordinates. As ANSYS is normalizing the normal modes to mass, the equation of motion in the principal coordinates will be of the form [4]:

$$\ddot{x}_{pi}(t) + 2 \cdot \xi_i \cdot \omega_i \cdot \dot{x}_{pi} + \omega_i^2 \cdot x_{pi}(t) = F_{pi}(t) \quad (4)$$

With $i=1, 2, \dots, n$ and

$$F_p(t) = [x_n]^T \cdot F(t) \quad (5)$$

being the vector of forces in principal coordinates system. As it follows, a set of n uncoupled differential equations of the second order is obtained from the set of n coupled differential equations of the second order. The resulting equations are converted into a state space form as first order differential equations. A general algorithm used for vibration analysis with ANSYS and MATLAB is summarized below and it was developed in [6].

4. MATHEMATICAL MODEL EXTRACTION TECHNIQUE

The state space model of the studied object can be constructed from the eigenvectors and eigenvalues normalized with respect to mass.

$$\begin{aligned} \dot{x} &= A \cdot x + B \cdot u \\ y &= C \cdot x + D \cdot u \end{aligned} \quad (6)$$

where \dot{x} it is state vector, and y it is output vector.

The system matrix A, input matrix B, output matrix C and direct transmission matrix D of the state space model for a dynamic system with n modes can be written as below [6]:

$$A = \begin{bmatrix} 0 & 1 & \dots & \dots & \dots \\ -\omega_1^2 & -2 \cdot \xi_1 \cdot \omega_1 & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & -\omega_1^2 & -2\xi_n \cdot \omega_n \end{bmatrix} \tag{7}$$

$$B = \begin{bmatrix} 0 \\ F_{p1} \\ 0 \\ F_{p2} \\ \dots \\ F_{pn} \end{bmatrix} \tag{8}$$

Here F_p is the vector of forces in principal coordinates. Value of C depends upon the output of interest so that for velocity output, C is given by:

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & \dots \\ 0 & x_{n11} & 0 & x_{n12} & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & x_{n21} & 0 & x_{n22} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix} \tag{9}$$

Having x_{n11}, x_{n12} the elements of the modal matrix normalized to mass.

For this analysis, no direct transmission to the output is considered hence:

$$D = [0] \tag{10}$$

Considering the above definitions of the state space matrices, having the eigenvectors and eigenvalues from ANSYS the state space system was constructed.

5. SIMULINK MODEL DEFINITION

Since the matrices representing the state space system are large for simplicity purposes, the state space system was converted to a transfer function with the aid of MATLAB.

The vibration simulations were performed considering an input force at the middle of the thin plate, node 497 and measured at node 1317 at the tip (the position of both nodes is indicated in *figure 3*). It is worth noting the fixed support position which was considered the far left edge of the plate, indicated in *figure 3* with 2 red arrows.

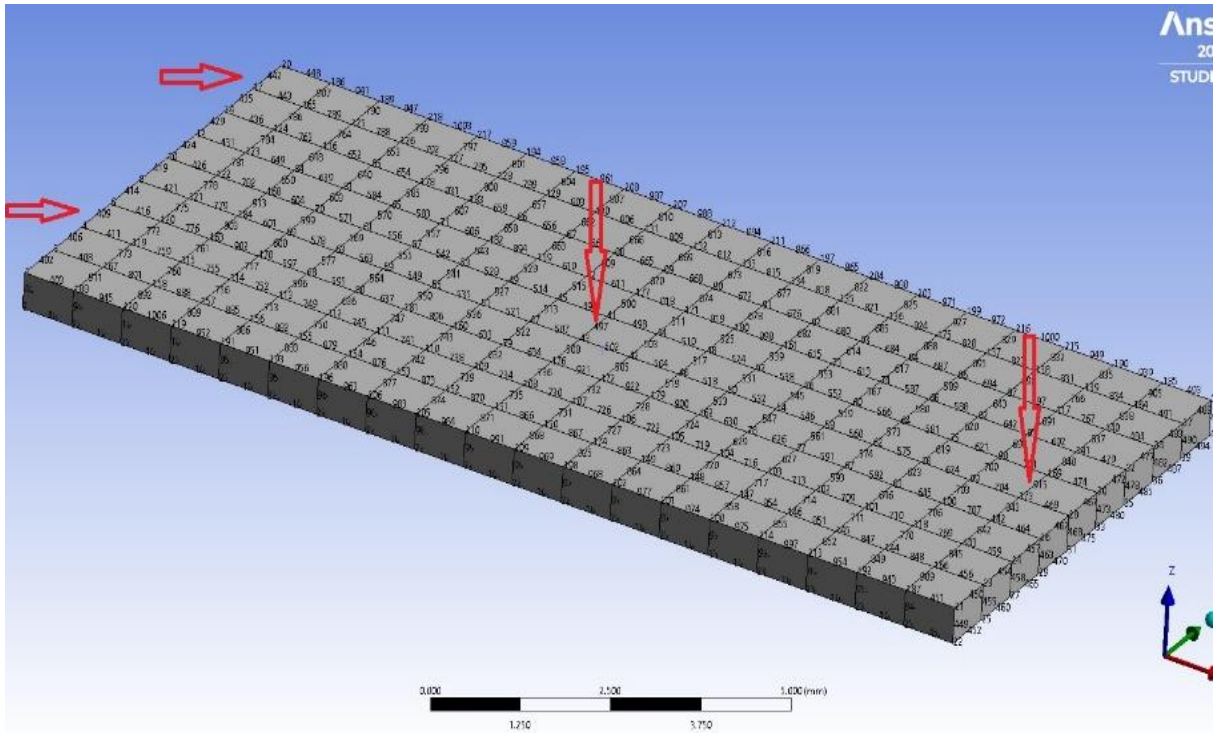


Fig. 3. Fixed support, input and output node position indication

The actuator model used in the SIMULINK system is a piezoelectric actuator defined and developed by another researcher in [1] and it is represented by the below transfer function.

$$TF_{act} = \frac{0.07962 \cdot s + 0.1521}{s^4 + 2.201 \cdot s^3 + 7.816 \cdot s^2 + s + 6.93} \tag{11}$$

The parameters used for the PID controller are listed in Table 3.

Table 3. PID controller parameters

Parameter	P	I	D	N
Value	49811618.8789	69870496.6521	8709427.0764	154.4799

The above defined components were integrated into a full SIMULINK model which is presented in *figure 4* below:

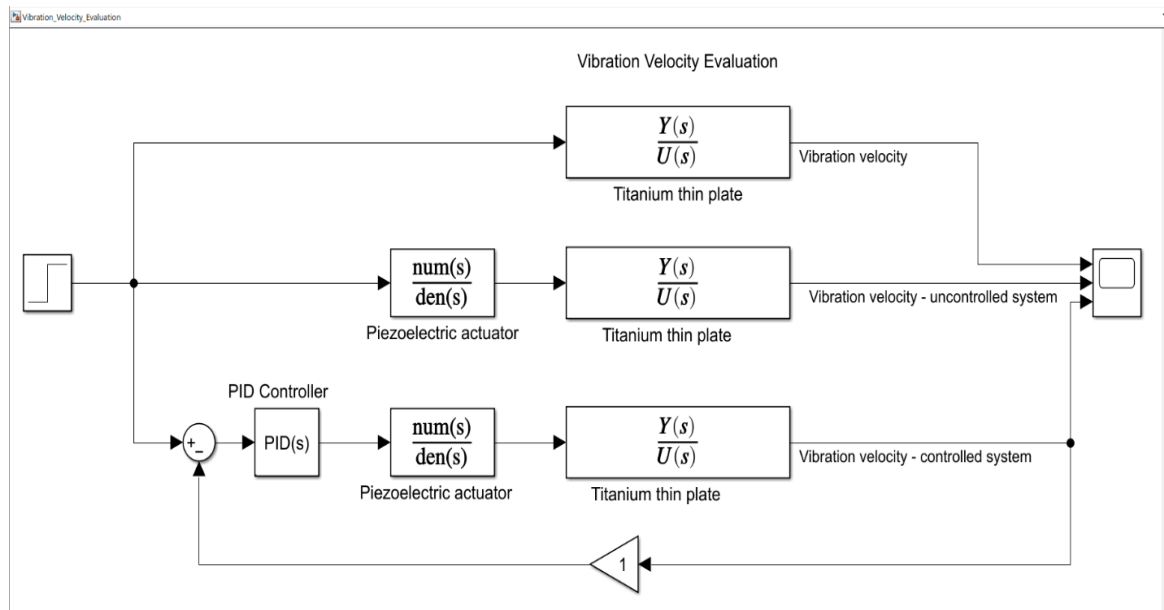


Fig. 4 The block diagram of the control system

6. RESULTS AND DISCUSSIONS

The uncontrolled system response to a unit force can be seen in *figure 5* below, over an interval of 0.5 seconds. The vibration velocity in this situation is oscillating between -0.6 to 0.6 m/s and it is never stabilizing over 0, oscillating continuously between -0.05 and 0.05 m/s. The system reason for which the system is never stabilizing is that it is considered an undamped system having the damping coefficient is 0.001.

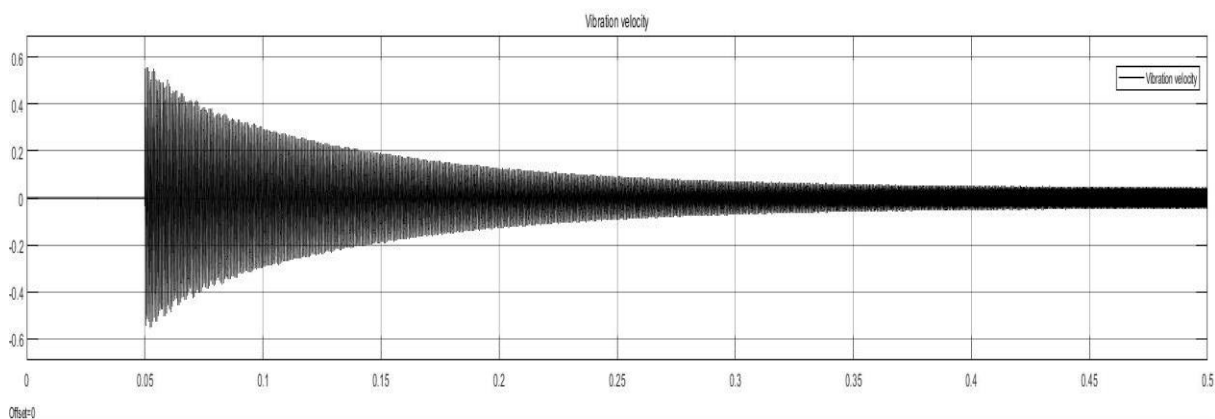


Fig. 5. Uncontrolled system vibration velocity

After introducing the piezoelectric actuator in the system, it can be observed a significant decrease in the vibration velocity (*fig 6*), however the introduction of a control system is needed. The system observation in this case was performed over a time interval of 10 seconds.

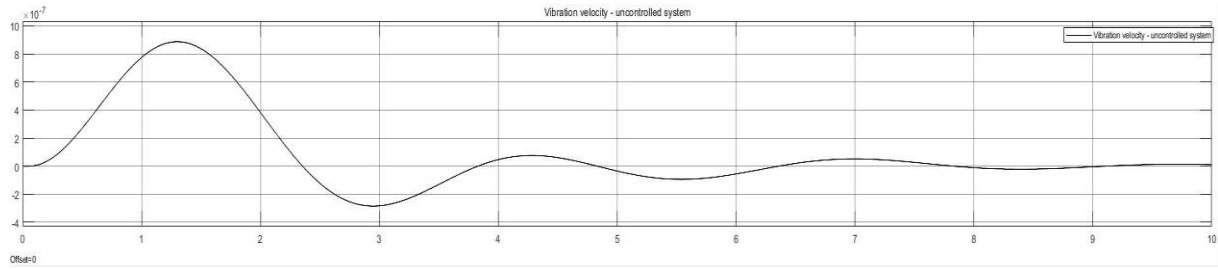


Fig. 6 The velocity of vibration after the introduction of the piezoelectric actuator

The 7th figure of this paper is presenting the system response after the introduction of the previously defined PID controller. The observation period is of 1 sec. As it can be observed, the system is stable, having the control curve parameters defined by a rise time of 0.0468 seconds, setting after 0.916 seconds with a peak amplitude of 1.12 and overshoot 12.1%.

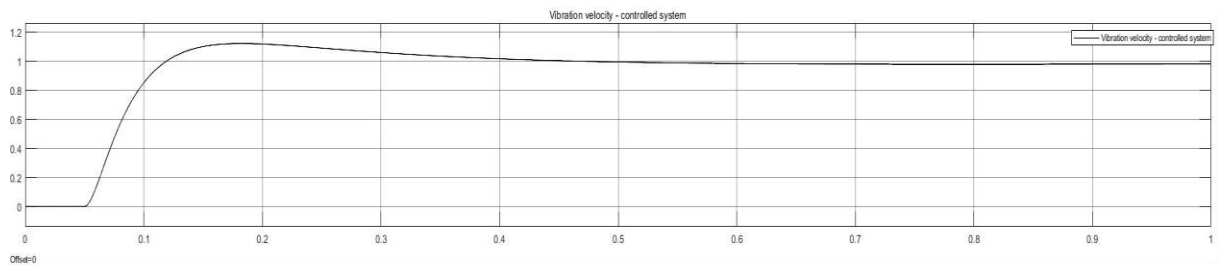


Fig. 7 Complete system response to step signal

The 8th figure is presented with the intent to show that the system is stabilizing in the proposed configuration after applying a fixed pulse signal at the input.

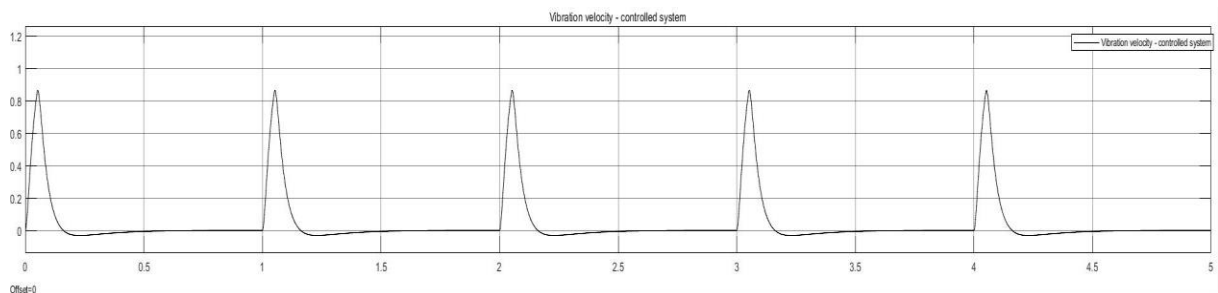


Figure 8. Complete system response to impulse signal

From what can be observed in *figure 9*, the controller is settling at the value of 0.9837 with an error of 0.0163 and as detailed in the description of the 7th figure, we can also mention that an efficient controller design was defined for this specific application.

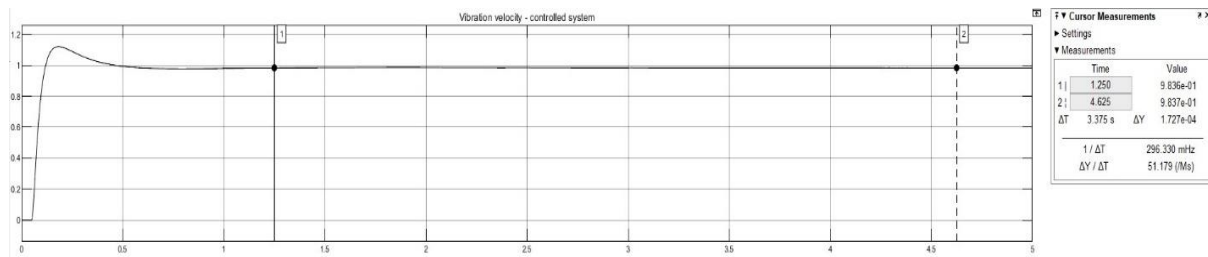


Fig. 9. Controller error definition

7. CONCLUSIONS

This paper presents the results of multiple simulations performed with ANSYS and MATLAB/SIMULINK software in order to evaluate the vibration of a thin titanium plate and the efficiency of a control system comprised of a piezoelectric actuator and PID controller.

In the images presented in *figures 2.1 – 2.6* the dynamics of the studied object can be observed and can be concluded that all the extracted modes of vibration can have a correspondent in reality as the shapes of vibration have understandable dynamic behavior.

In chapter 6, various SIMULINK simulation results are presented, and the efficiency of the control system can be observed. The vibration reduction is significantly reduced, and the controller output characteristics are showing good performances.

After analyzing the results of the simulations, we can conclude that the algorithm used to define the mathematical model of the mechanical object is found to be reliable and effective in performing an initial selection of a control system plant model. The results are inspiring and encouraging to apply the presented technique also for other type of applications.

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