CONSIDERATIONS ABOUT ELECTRODYNAMIC FORCES ANALYTICAL COMPUTATION

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Abstract*: The electrodynamic forces deped on the strength of the currents and conductors shapes and mutual positions. For simple configurations are available analytical solutions but for complex ones only numerical methods could be used. Anyway only the real-life tests will quarantee the accuracy in design process. So, the fastest method to predict the electrodynamic forces with acceptable error is desired. This paper deals with analytical solutions and the availability of each one regarding the imposed precision. The influence of filiformity and the infinite length is studied.*

1. INTRODUCTION

Electrodynamic forces are forces acting between two carying currents conductors or between a conductor and a magnetic field. There are three methods to compute the forces [1 - 3]:

- ✓ Laplace’s force,
- ✓ Virtual work method,
- ✓ Maxwell’s stress method.

1.1. Laplace’s force

Based on Laplace’s law, on an element *dl* of a circuit, through which a current of strength *i* flows, placed in a magnetic field *B*, an electrodynamic forces is exercited:

$$d\bar{f} = i \cdot d\bar{l} \times \bar{B} \tag{1}$$

To calculate the magnetic field, in a point situated at distance R from an element of circuit, *dl*, wich produces the field, Biot-Savarat's law could be applied:

$$d\bar{H} = \frac{1}{4\pi} \cdot \frac{i \cdot d\bar{l} \times \bar{R}}{R^3} \quad (2)$$

The Ampere's theorem also lead to the magnetic field in a point, generated by a current flowing through a conductor crossing any surface S bounded by the of contour C.

$$\oint_C \bar{B} \cdot d\bar{l} = \mu_0 \iint_S \bar{J} \cdot d\bar{s} = \mu_0 \cdot i \quad (3)$$

1.2. Virtual work method

According to the virtual work principle, the force exercited by in a physical system can be computed based on stored magnetic co-energy, W_{co}, change due a small displacement, *x*:

$$F = -\left.\frac{\partial W_{co}}{\partial x}\right|_{\Phi=cst.} \quad (4)$$

or:

$$F = \left.\frac{\partial W_{co}}{\partial x}\right|_{i=cst.} \quad (5)$$

For a very small variation of *x* the differential operator could be replaced by simple extraction:

$$F = -\left.\frac{\Delta W_{co}}{\Delta x}\right|_{\Phi=cst.} \quad (6)$$

or:

$$F = \left.\frac{\Delta W_{co}}{\Delta x}\right|_{i=cst.} \quad (7)$$

1.3. Maxwell's stress method

The third method for electrodynamic forces calculation is one of the most used in numerical analysis, especially in Finite Element Method postprocessing. The use of Maxwell's stress method asked an integration of the component of the stress over a surface passing entirely trough air:

$$\bar{F} = \oint_S \left(\bar{H}\left(\bar{B}\bar{n}\right) - \frac{1}{2}\left(\bar{H}\bar{B}\right)\bar{n} \right) dS \quad (8)$$

For a given configuration, one of the above presented method will be less difficult to be applied then others. However all approaches should produce the same result, but the small differences occurring are due to assumptions considered for each ones.

2. TWO PARALLEL FILLIFORM RECTILINEAR CONDUCTORS

2.1. Infinite lenght

For this configuration, *fig. 1*, relations (1) and (2) lead to one of the most known elecrodynamic force formulae, named Ampere's force:

$$F_{12} = F_{21} = \frac{\mu_0}{2\pi} i_1 \cdot i_2 \frac{l}{d} = 2 \cdot 10^{-7} \cdot i_1 \cdot i_2 \frac{l}{d} \tag{9}$$

Force F_{12} is the force exercited by conductor 2 on conductor 1 and viceversa.

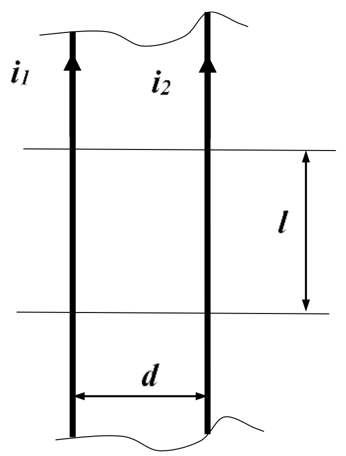

Fig. 1. Two parallel filiform rectilinear infinite length conductors

Relation (9) is a very simple one and it is important to use it for as much configurations as it is possilble. In the next paragraphs, the influence of the finite length and the shape of conductors will be studied.

2.2. Influence of the finite lenght

In real-life, conductors have finite length. If the lengths of conductors are equal and the conductors are spaced as in *fig. 2*, relation (9) has to be altered by a length function *C(d/l)*, [3-4]:

$$C = \sqrt{1 + \frac{d^2}{l^2}} - \frac{d}{l} \tag{10}$$

The force will be computed as:

$$F = \frac{\mu_0}{2\pi} i_1 \cdot i_2 \frac{l}{d} \cdot C = 2 \cdot 10^{-7} \cdot i_1 \cdot i_2 \frac{l}{d} \cdot C \tag{11}$$

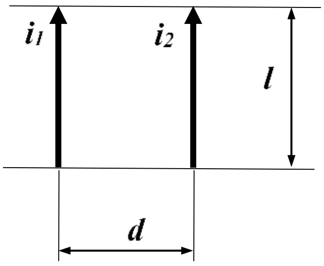

Fig. 2. Two parallel filiform rectilinear finite equal length conductors

Compareing relations (9) and (11) it is import to outline the error produced for a given arranjaments of the conductors. This could be easelly done trough a simple graphical representation of the function *C(d/l)*.

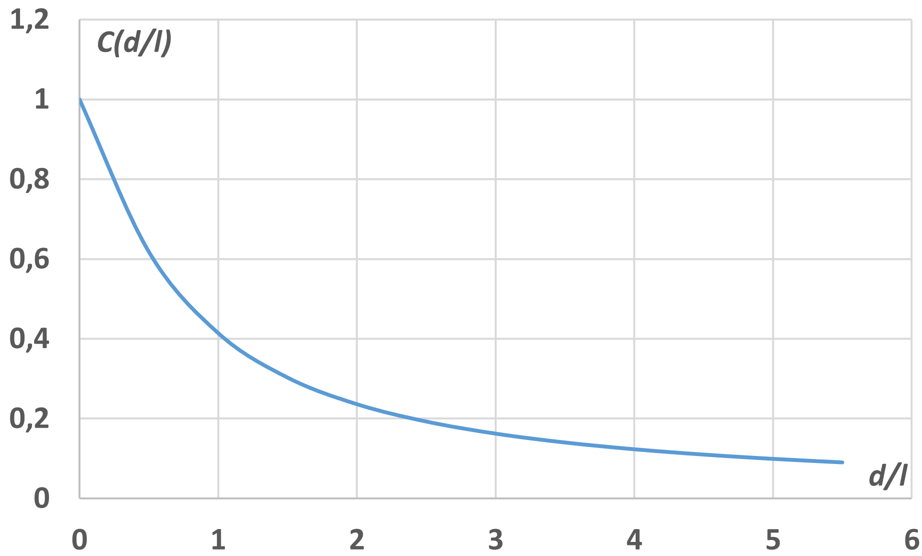

Fig. 3. C(d/l) graphical representation

Also if the relative error is considered:

$$\varepsilon = \frac{F_{finite} - F_{\inf inite}}{F_{finite}} \cdot 100[\%] \tag{12}$$

with F_{finite} being the force computed with (11) and $F_{infinite}$ the Ampere's force (9), it is simple to set up an desired error and find the appropriate assumption of C that fulfill it. Applying in (12) relations (9) and (11) will get:

$$\varepsilon = \frac{C-1}{C} \cdot 100 = \left[1 - \frac{1}{C}\right] \cdot 100 \tag{13}$$

It is obviouslly that the error is negative, the force per length is bigger for an infinite length conductors than for finite length conductors.

From (13), the ratio *d/l* goes to:

$$\frac{d}{l} = -\frac{1}{2} \cdot \frac{\varepsilon \cdot (200-\varepsilon)}{100 \cdot (100-\varepsilon)} \tag{14}$$

The graphical interpretation of (14) for an error between 0 and -10%, *fig. 4,* shows a very intuitive way of limitation of Ampere's force when finite length conductors are involved. E.g. for a desired error bellow 5% the ratio *d/l* must be above 1/20, meaning a length of 20 times bigger than the distance between conductors and 1% weeoe leads to a $d/l \geq 1/100$.

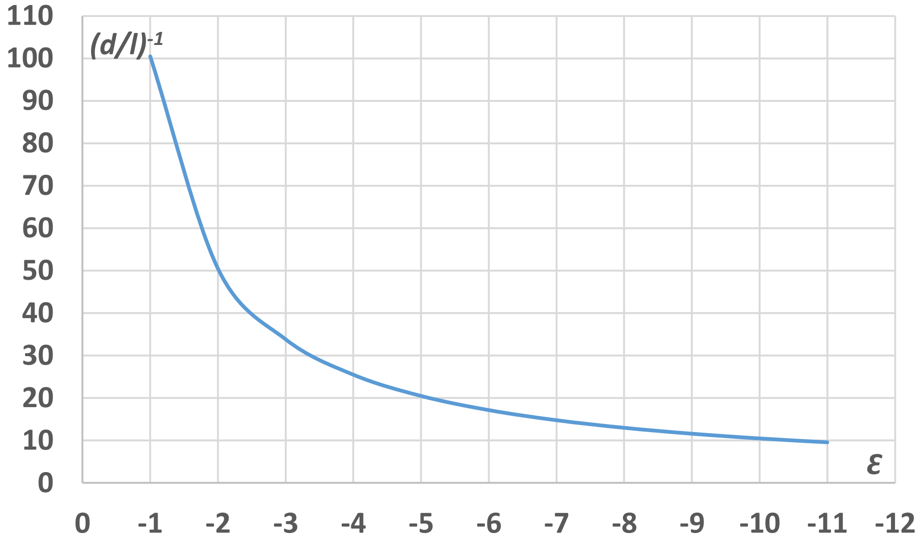

Fig. 4. $(d/l)^{-1}$ in terms of ε, graphical representation

For two parallel filiform rectilinear finite unequal length conductors, *fig. 5,* relation (11) must be completed with two lenght functions, denoted C_1 and C_2, [4]:

$$F = \frac{\mu_0}{2\pi} i_1 \cdot i_2 \frac{l}{d} \cdot (C_1 + C_2) = 2 \cdot 10^{-7} \cdot i_1 \cdot i_2 \frac{l}{d} \cdot (C_1 + C_2) \tag{15}$$

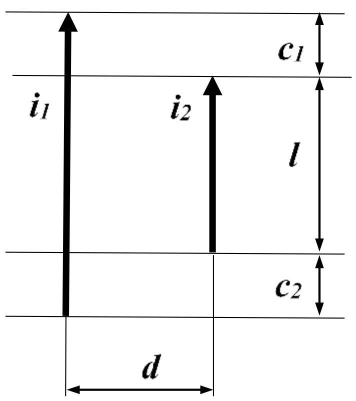

Fig. 5. Two parallel filiform rectilinear finite unequal length conductors

The values of C_1 and C_2 could be computed using (16) and (17), [4] or can be read from *fig. 6* [4]:

$$C_1 = \sqrt{\left(1+\frac{c_1}{l}\right)^2 + \frac{d^2}{l^2}} - \sqrt{\frac{c_1^{\ 2}}{l^2} + \frac{d^2}{l^2}} \tag{16}$$

$$C_2 = \sqrt{\left(1+\frac{c_2}{l}\right)^2 + \frac{d^2}{l^2}} - \sqrt{\frac{c_2^{\ 2}}{l^2} + \frac{d^2}{l^2}} \tag{17}$$

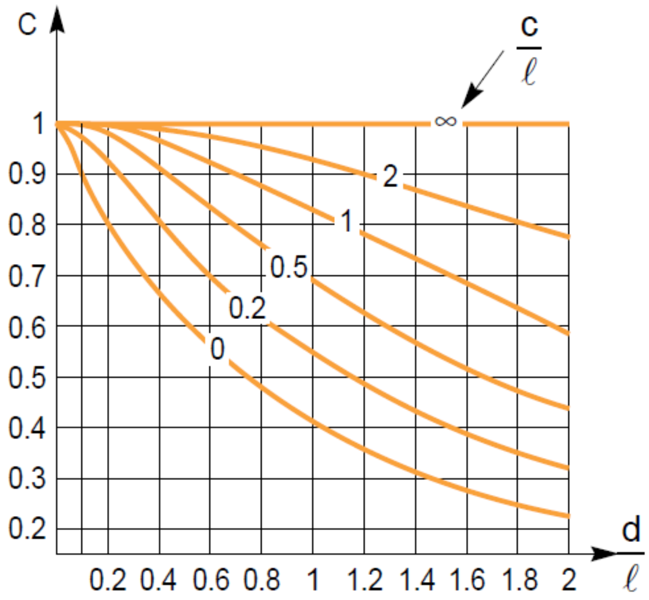

Fig. 6. C(c/l, d/l) graphical representation

As it can be easlly seen, for $c_1 = 0$, $C_1 = f(d/l)$ from (10). For simplicity the value of *f(d/l)* can be selected, for a given *d/l*, from *fig. 6*, choosing *c/l* = 0.

Anyway, for an existing configurations of conductors, the value of lenght function C_1+C_2 or C can be computed in term of the error, from (13), as:

$$C = \frac{100}{100-\varepsilon} \tag{18}$$

$$C_1 + C_2 = \frac{100}{100-\varepsilon} \tag{19}$$

Imposing an error of -5%, goes to a value of lenght factor equal to 0.95. Supposing that $c = 0$, from fig. 6 the requested *d/l* is close to 0.05, meaning the same result as from (14) or *fig. 4*.

2.3. Influence of conductor shape

For conductors having specific shapes, i.e. non filiforms, the electrodynamic forces are computed based on virtual work principle.

For configuration shown in *fig. 7*, reassumed infinite lengths, the force is, [3]:

$$F = \frac{\mu_0}{2\pi} i_1 \cdot i_2 \frac{l}{(d-r)} = 2 \cdot 10^{-7} \cdot i_1 \cdot i_2 \frac{l}{(d-r)} = 2 \cdot 10^{-7} \cdot i_1 \cdot i_2 \frac{l}{d} \cdot k \tag{20}$$

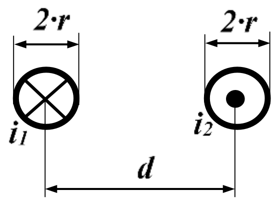

Fig. 7. Two parallel circular infinite length conductors

In (20) *k* is a shape function, used to maintain the simplest relation in force calculation:

$$k = \frac{d}{(d-r)} = \frac{1}{1 - r/d} \tag{21}$$

The graphical representation of (21) is shown bellow.

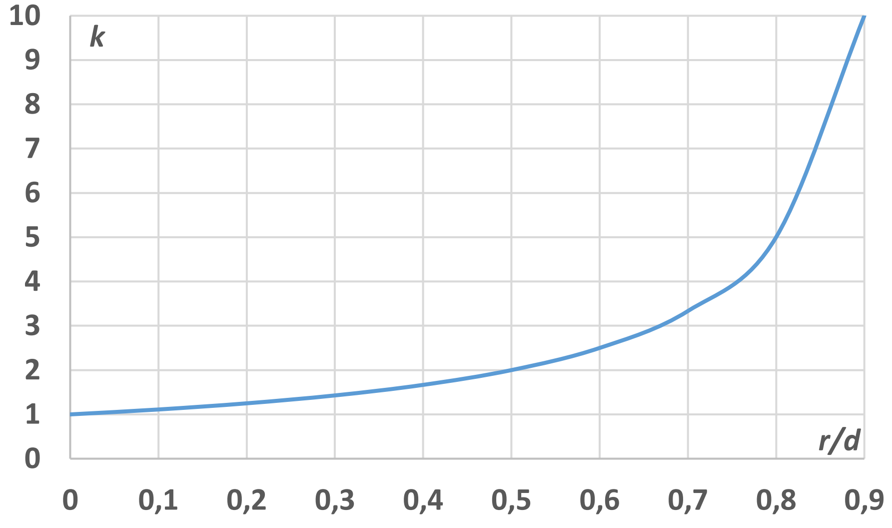

Fig. 8. k(r/d) graphical representation

Considering, again, the relative error regarding the Ampere's force:

$$\varepsilon = \frac{F_{shape} - F_{filiform}}{F_{shape}} \cdot 100 [\%], \tag{22}$$

it is easy to find the ratio between the conductor radius and the distance between the wires that fulfill an imposed value of error:

$$\frac{r}{d} = \frac{\varepsilon}{100} \tag{23}$$

For a desired error of 1% the distance between conductors must be 100 times bigger than the radius of the cross-section of the wire.

If the conductors have rectangular cross-sections, *fig. 9, upper right corner*, the shape function, denoted *k*, is more complex and depends by the actual position of conductors. To avoid complex calculus, k could be chooses from Dwight's chart, fig. 9, [4].

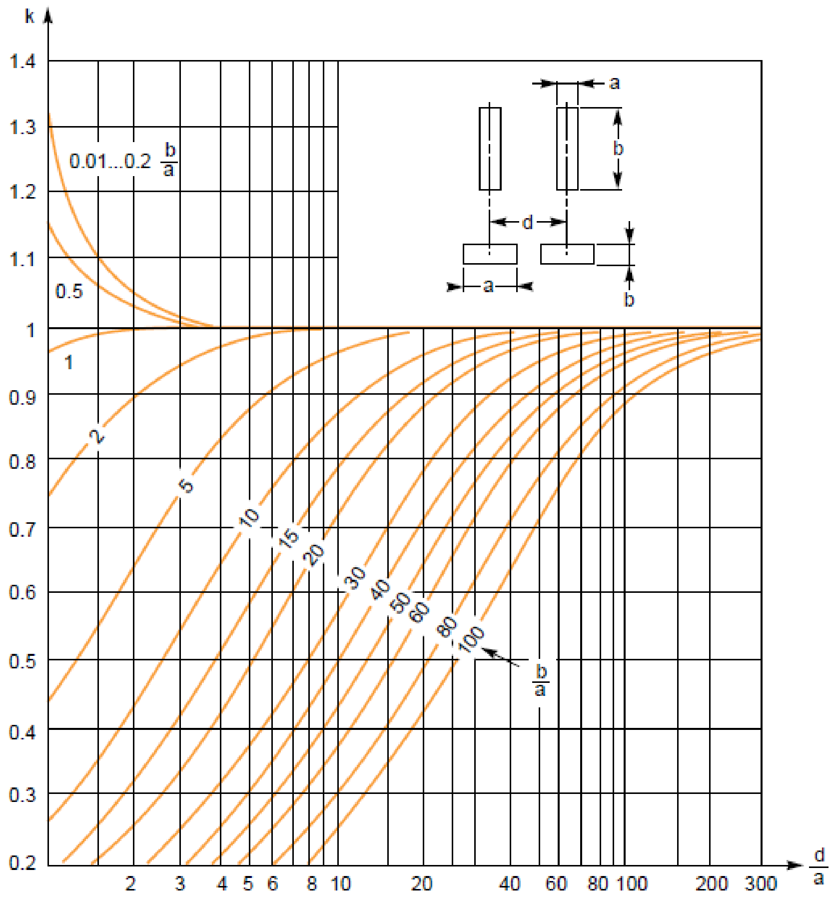

Fig. 9. Dwight's chart

The force exercited on conductors is:

$$F = \frac{\mu_0}{2\pi} i_1 \cdot i_2 \frac{l}{d} \cdot k = 2 \cdot 10^{-7} \cdot i_1 \cdot i_2 \frac{l}{d} \cdot k \tag{24}$$

Changing in (18) *C* with *k*, all remarks are applied also for shape function.

2.4. Influence of both conductor length and shape

Composing above presented configurations, a general relation for parallel wires configurations can be written:

$$F = \frac{\mu_0}{2\pi} i_1 \cdot i_2 \frac{l}{d} \cdot C \cdot k = 2 \cdot 10^{-7} \cdot i_1 \cdot i_2 \frac{l}{d} \cdot C \cdot k \tag{25}$$

for equal length, and, for unnequall length:

$$F = \frac{\mu_0}{2\pi} i_1 \cdot i_2 \frac{l}{d} \cdot \left(C_1 + C_2\right) \cdot k = 2 \cdot 10^{-7} \cdot i_1 \cdot i_2 \frac{l}{d} \cdot \left(C_1 + C_2\right) \cdot k \qquad (26)$$

Again, changing in (18) C with $C \cdot k$, or C with $(C_1 + C_2) \cdot k$ all remarks are applied also for combined length and shape functions.

3. CONCLUSIONS

In this paper have been presented some considerations about analitycal computation of the electrodynamic forces for parallel wires. All relations have been written based on Ampere’s force relation, because of simplicity and easy of use. All the length and shape functions are graphically interpreted and the limitation in Ampere’s force is depicted in term of relative error.

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