NUMERICAL EVALUATION OF SELF AND MUTUAL EARTH RETURN IMPEDANCES

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Abstract: *The paper presents an evaluation of analytical self and mutual impedances formulas of lines with earth return, taking into account the ground correction terms. The determined formulas contain semi-infinite integral terms which are calculated by a novel stable and efficient numerical integration scheme in order to overcome the involved oscillation problems. It might seek approximations of the semi-infinite integrals by replacing an exponential or algebraic function, the objective being to permit analytic integration. Since there is no good systematic method for making these replacements, their success depends directly on the intuition and ingenuity, taking into account that in practice the integrand has limited accuracy.*

1. INTRODUCTION

The problem of environmental effects and electromagnetic interferences produced by time varying electric and magnetic fields due to A.C. currents flowing through high voltage or medium voltage power lines have been the topic of numerous research papers over the last decades [1-3]. Therefore, it is well known that due to inductive and capacitive couplings, dangerous eddy currents could be induced in any metallic structure placed nearby overhead or underground power lines. These dangerous situations could be also detected in case of underground electrical cables or cable screens, designed to attenuate the electric and magnetic field distributions in the surrounding zone. The result consists in insulation overheating which leads to structural integrity of these cables [4, 5].

There are generally two ways to analyze the electromagnetic interference between transmission lines and nearby parallel metallic structures: (1) using the conventional circuit method along with grounding analysis or (2) using the electromagnetic field method. Regardless of the method used, an essential step during the numerical algorithm is the computational of the self and mutual impedances of the underground/aerial conductors.

There are many efforts to solve the problem in the reference materials, but usually the authors made the approximations derived from the circuit's theory. Many physical events cannot be taken into account using the elements with concentrated parameters, i.e. grounding resistors, capacitors and inductances to describe the behavior into the soil. To avoid mathematical difficulties, it is introduced some simplifications, which conceal the physical picture of the problem. Skin effect in the soil, especially for different earth's resistivity, is hidden in empiric formulas, diagrams and nomograms, usually applied in power engineering. For this reason, it has been decided to take the physical base as an essential electromagnetic starting-point and the applied mathematical methods used to compute the self and mutual impedances, will be only the consequence of such a treatment [6].

2. EARTH RETURN IMPEDANCE EVALUATION USING CARSON FORMULAS

J. R. Carson published a theory [7] suitable to calculate the electromagnetic fields due to a horizontal current carrying conductor, which is above a lossy earth. The integral found by Carson is usually applied for the calculation of self and mutual impedance of lines with earth return. He was one of the firsts who evaluated transmission line parameters and found expressions for the elements of the impedance matrix Z that nowadays serve as a good reference under the name "Carson's equations". Accordingly to this theory, the calculation of line impedance (self and mutual impedance of lines with earth return) is based on equations that contain semi-infinite integrals with complex arguments. For evaluation of these integrals, infinite series and also some convenient approximations for low frequencies have been proposed.

In the mutual impedance equation (1) could be identified a Sommerfeld integral [6, 8], highlighted in equation (2),

$$\underline{Z}_{12} = j2\omega \ln\left(\frac{\rho''}{\rho'}\right) + 4\omega \int_0^\infty \left(\sqrt{\mu^2 + j} - \mu\right) e^{-(h_k + h_l)\mu} \cos(x'\mu) d\mu \tag{1}$$

$$J(p,q) = \int_0^\infty \left(\sqrt{\mu^2 + j} - \mu\right) \cdot e^{-p\mu} \cdot \cos(q\mu) \cdot d\mu \tag{2}$$

which formulates a solution J = P + jQ in the form of a sum of eight infinite series (*figure 1*):

$$\begin{cases} P = \dfrac{\pi}{8}\left(1 - s_4\right) + \dfrac{1}{2}\ln\left(\dfrac{2}{\gamma r}\right)\cdot s_2 + \dfrac{1}{2}\theta \cdot s'_2 - \dfrac{1}{\sqrt{2}}\sigma_1 + \dfrac{1}{2}\sigma_2 \, \dfrac{1}{\sqrt{2}}\sigma_3 \\ Q = \dfrac{1}{4} + \dfrac{1}{2}\ln\left(\dfrac{2}{\gamma r}\right)\cdot\left(1 - s_4\right) - \dfrac{1}{2}\theta \cdot s'_4 + \dfrac{1}{\sqrt{2}}\sigma_1 - \dfrac{\pi}{8}s_2 + \dfrac{1}{\sqrt{2}}\sigma_3 - \dfrac{1}{2}\sigma_4 \end{cases} \tag{3}$$

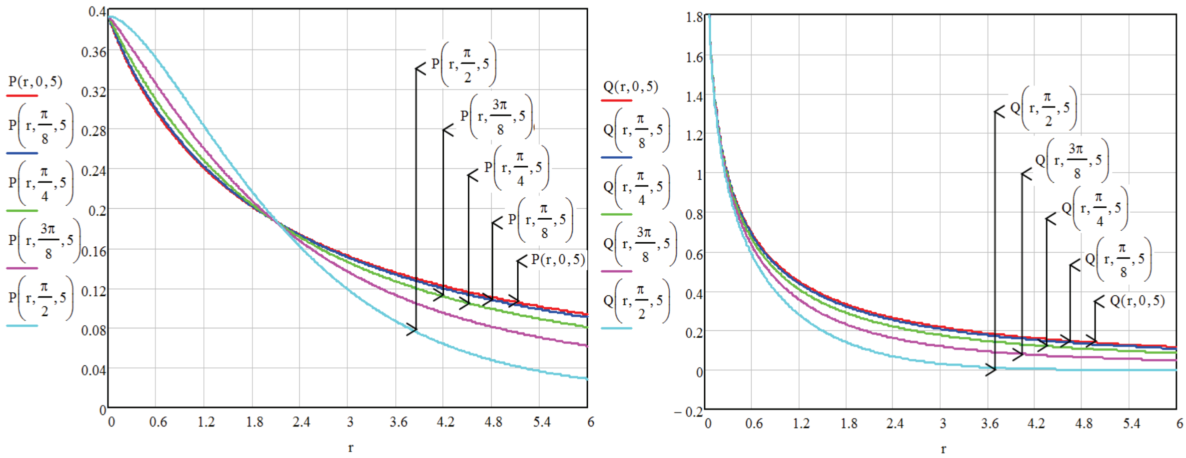

Fig. 1 – Representation of the P and Q functions

The infinite series are rapidly convergent and an implementation in a numerical computation software lead to the function J, plotted against r and θ and using n = 5 as the number of considered factors [9].

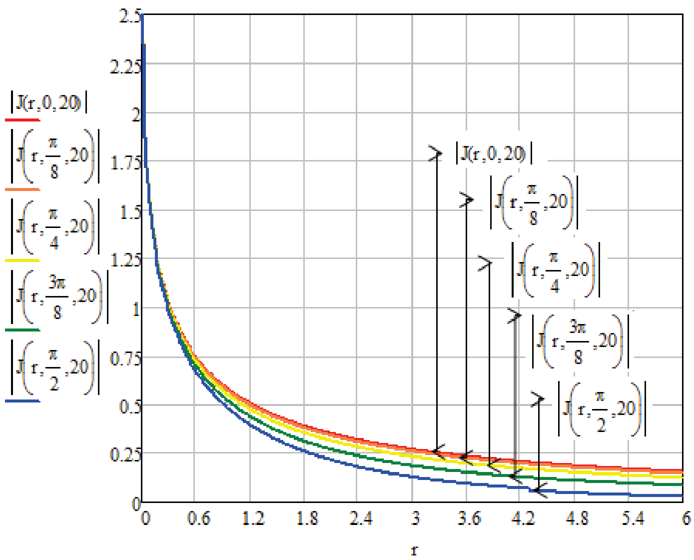

Fig. 2 - Function J plotted against r and θ

Introducing in inductances relations the (concept of) skin depth these parameters becomes a complex numbers with terms which represent the energy dissipation in the ground. This principle is available for conductor's self and mutual inductance calculations [10].

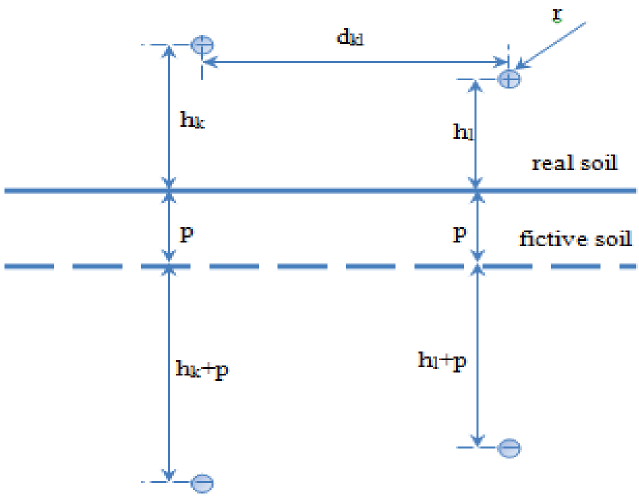

Fig. 3 – Geometrical configuration using a fictive ground return plane

The complex ground return plane could be explained using some fundamental equations for self and mutual impedances of the ideal conductor (complex plane). It starts with the ideal conductor/ground return (self and mutual) impedances:

$$\begin{cases} \underline{Z}_m = j\omega \dfrac{\mu_0}{2\pi} \ln \dfrac{\sqrt{(h_k + h_l)^2 + d_{kl}{}^2}}{\sqrt{(h_k - h_l)^2 + d_{kl}{}^2}} + \underline{Z}_{mg} \\ \underline{Z}_s = j\omega \dfrac{\mu_0}{2\pi} \ln \dfrac{2h}{r} + \underline{Z}_{sg} \end{cases} \quad (4)$$

The impedances Z_{mg}, Z_{sg} are the Carson's ground correction terms of the mutual and self-impedance per unit length that introduce the skin depth effect in the ground.

Could be recognizing in the first term of the equations (4), with the effect hypothesis (the alternative current is concentrating on the conductor exterior surface) [10]. The ground correction term of the impedances per unit length is:

$$\underline{Z}_{mg} = j\omega \frac{\mu_0}{2\pi} \int_0^{\infty} \frac{2e^{-(h_k + h_l)\lambda}}{\lambda + \sqrt{\lambda^2 + j\omega\mu_0\sigma}} \cdot \cos(\lambda d_{kl}) \cdot d\lambda \quad (5)$$

$$\underline{Z}_{sg} = j\omega \frac{\mu_0}{2\pi} \cdot \int_0^{\infty} \frac{2e^{-2h\lambda}}{\lambda + \sqrt{\lambda^2 + j\omega\mu_0\sigma}} \cdot d\lambda \quad (6)$$

After evaluating the semi-infinite integrals using series expansion it is obtained the following expressions of impedances, using the complex earth return plane:

$$\begin{cases} \underline{Z}_s = j\omega \dfrac{\mu_0}{2\pi} \cdot \ln\left(\dfrac{2(h+p)}{r}\right) \\ \underline{Z}_m = j\omega \dfrac{\mu_0}{2\pi} \cdot \ln\left(\dfrac{\sqrt{(h_k + h_l + 2p)^2 + d_{kl}{}^2}}{\sqrt{(h_k - h_l{}^2) + d_{kl}{}^2}}\right) \end{cases} \quad (7)$$

These equations provide simple and also remarkably accurate substitutes to Carson's equations (electronic handheld calculators can evaluate them instead of computers) over the whole range of frequencies.

The authors developed an algorithm which can compute the error between the formulas for self and mutual impedances, for arbitrary geometries (see the results in *figure 4*) with the following steps (*figure 3*):

STEP 1:

$$J_{Carson}(f,k,l) := \begin{cases} \displaystyle\int_0^{\infty} \frac{j\cdot e^{-2h_k\cdot\lambda}}{\lambda+\sqrt{\lambda^2+j\cdot\omega(f)\cdot\mu_0\cdot\sigma}}\,d\lambda & \text{if } k=1 \\ \displaystyle\int_0^{\infty} \frac{j\cdot e^{-2(h_k+h_l)\cdot\lambda}}{\lambda+\sqrt{\lambda^2+j\cdot\omega(f)\cdot\mu_0\cdot\sigma}}\cdot\cos(\lambda\cdot d_{k,l})\,d\lambda & \text{otherwise} \end{cases}$$

$$Z_{Carson}(f,k,l) := \begin{cases} j\cdot\omega(f)\cdot\dfrac{\mu_0}{2\pi}\cdot\ln\left(\dfrac{2\cdot h_k}{r}\right)+\omega(f)\cdot\dfrac{\mu}{\pi}\cdot J_{Carson}(f,k,l) & \text{if } k=1 \\ j\cdot\omega(f)\cdot\dfrac{\mu_0}{2\pi}\cdot\ln\left[\dfrac{\sqrt{(h_k+h_l)^2+(d_{k,l})^2}}{\sqrt{(h_k-h_l)^2+(d_{k,l})^2}}\right]+\omega(f)\cdot\dfrac{\mu}{\pi}\cdot J_{Carson}(f,k,l) & \text{otherwise} \end{cases}$$

$$R_{Carson}(f,k,l) := \mathrm{Re}(Z_{Carson}(f,k,l))$$
$$X_{Carson}(f,k,l) := \mathrm{Im}(Z_{Carson}(f,k,l))$$

STEP 2:

$$Z_{prop}(f,k,l) := \begin{cases} j\cdot\omega(f)\cdot\dfrac{\mu_0}{2\pi}\cdot\ln\left(\dfrac{2\cdot h_k+p(f)}{r}\right) & \text{if } k=1 \\ j\cdot\omega(f)\cdot\dfrac{\mu_0}{2\pi}\cdot\ln\left[\dfrac{\sqrt{(h_k+h_l+2p(f))^2+(d_{k,l})^2}}{\sqrt{(h_k-h_l)^2+(d_{k,l})^2}}\right] & \text{otherwise} \end{cases}$$

$$R_{prop}(f,k,l) := \mathrm{Re}(Z_{prop}(f,k,l))$$
$$X_{prop}(f,k,l) := \mathrm{Im}(Z_{prop}(f,k,l))$$
$$D(k,l) := \sqrt{(d_{k,l})^2+(h_k+h_l)^2}$$
$$a(f,k,l) := \frac{\sqrt{2}}{|p(f)|}\cdot D(k,l)$$

STEP 3:

$$\varepsilon_R(f,k,l) := \frac{R_{Carson}(f,k,l)-R_{prop}(f,k,l)}{|Z_{Carson}(f,k,l)|} \qquad \varepsilon_X(f,k,l) := \frac{X_{Carson}(f,k,l)-X_{prop}(f,k,l)}{|Z_{Carson}(f,k,l)|}$$

$$\varepsilon_R(f,k,l) := \frac{R_{Carson}(f,k,l)-R_{prop}(f,k,l)}{|Z_{Carson}(f,k,l)|} \qquad \varepsilon_Z(f,k,l) := \frac{|Z_{Carson}(f,k,l)|-|Z_{prop}(f,k,l)|}{|Z_{Carson}(f,k,l)|}$$

Fig. 4 – Error computation using a Mathcad algorithm

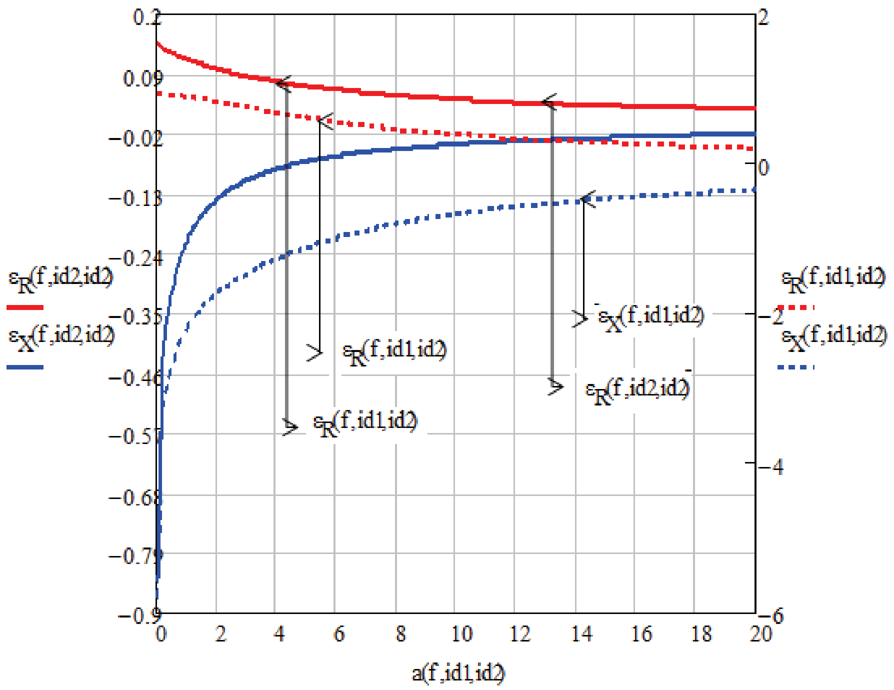

Fig. 5 - Relative error between the formulas for self and mutual impedances, for arbitrary geometries

3. EARTH RETURN IMPEDANCE EVALUATION USING POLLACZEK FORMULAE

Pollaczek [11] derives a solution for the electromagnetic coupling problem by following roughly the same path as Carson. His solution is also formulated in an integral from, however this integral is a doubly infinite one compared to Carson's semi-infinite one. To calculate buried cable earth impedances, one has to solve Pollaczek's integral, which, as the one by Carson, does not have a closed form solution.

But, whilst Carson's integral can be solved numerically with relative easiness (using the infinite series expansion), Pollaczek's integrand is highly oscillatory and irregular. For this reason, its integration by series or by general purpose algorithms presents convergence problems. Wedepohl [12] has proposed a series solution to Pollaczek's integral, but this series is very difficult to handle and has slow convergence rate at certain application ranges [13].

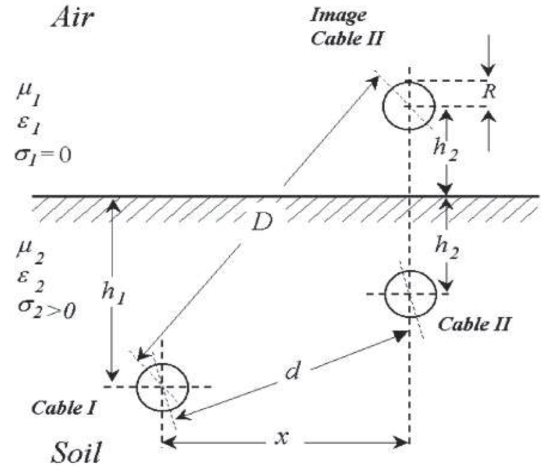

Fig. 6 - Underground transmission network

For a system of two underground cables, presented in *figure 6*, the self and mutual impedances can be calculated with equation (8)

$$\underline{Z}_P = \frac{j\omega\mu_0}{2\pi}\left[K_0\left(\frac{d}{p}\right) - K_0\left(\frac{D}{p}\right) + J\right] \tag{8}$$

where J is the infinite integral and has the form emphasized in equation (9):

$$J = \int_{-\infty}^{+\infty} \frac{\exp\left[-2\cdot h\sqrt{\beta^2 + \frac{1}{p^2}}\right]}{|\beta| + \sqrt{\beta^2 + \frac{1}{p^2}}} \cdot \exp(j\beta x)\cdot d\beta \tag{9}$$

By introducing in equation (8) the variable change: $\beta = u/|p|$ respectively considering that $\sqrt{u^2 + j} + u = \frac{j}{\sqrt{u^2 + j} - u}$, a new formula for J is obtained:

$$J=-2j\cdot\int_0^\infty\left(\sqrt{u^2+j}-u\right)\cdot\exp\left[-\left(\frac{2h}{|p|}\right)\sqrt{u^2+j}\right]\cdot\cos\left(u\frac{x}{|p|}\right)\cdot du \tag{10}$$

It should be noted that h and x are always divided by $|p|$, which is interpreted as normalizations of distances h and x with respect to the penetration depth $|p|$. Thus, the following reduced dimensions parameters are introduced in equation (10): $\xi=\dfrac{2h}{|p|}$, $\eta=\dfrac{x}{2h}$, alongside the notation:

$$\sqrt{u^2+j}=\frac{\sqrt{u^2+\sqrt{u^4+1}}}{\sqrt{2}}+j\cdot\frac{\sqrt{-u^2+\sqrt{u^4+1}}}{\sqrt{2}}=F(u)+j\cdot G(u) \tag{11}$$

in order to obtain a final formula for *J,* that highlights the fact that this integral is a function of dimensionless variables ξ and η only:

$$J=-2j\cdot\int_0^\infty[F(u)-u+j\cdot G(u)]\cdot\exp\{-\xi[F(u)+jG(u)]\}\cdot\cos(\xi\eta u)\cdot du \tag{12}$$

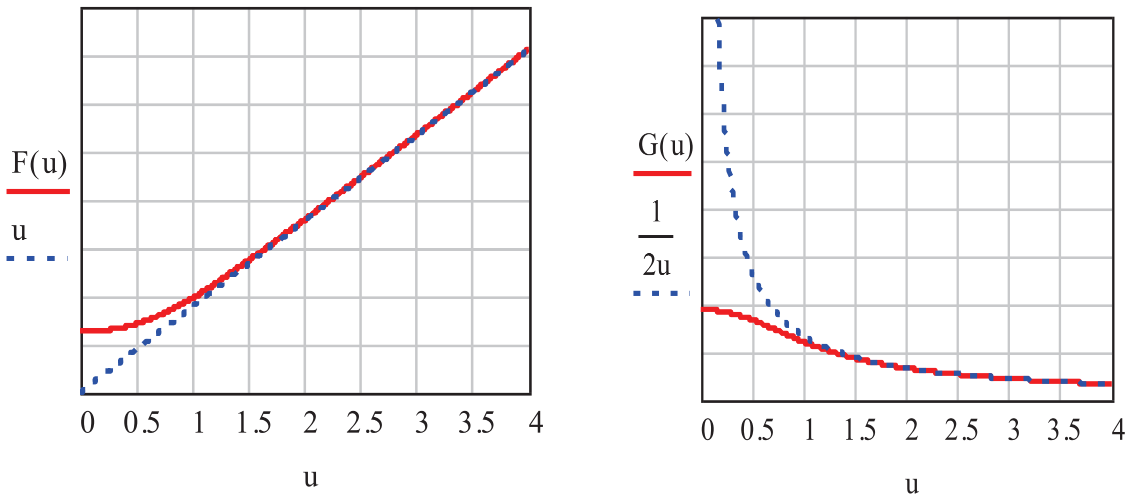

Fig. 7 - Graphical representation of F(u) and G(u)

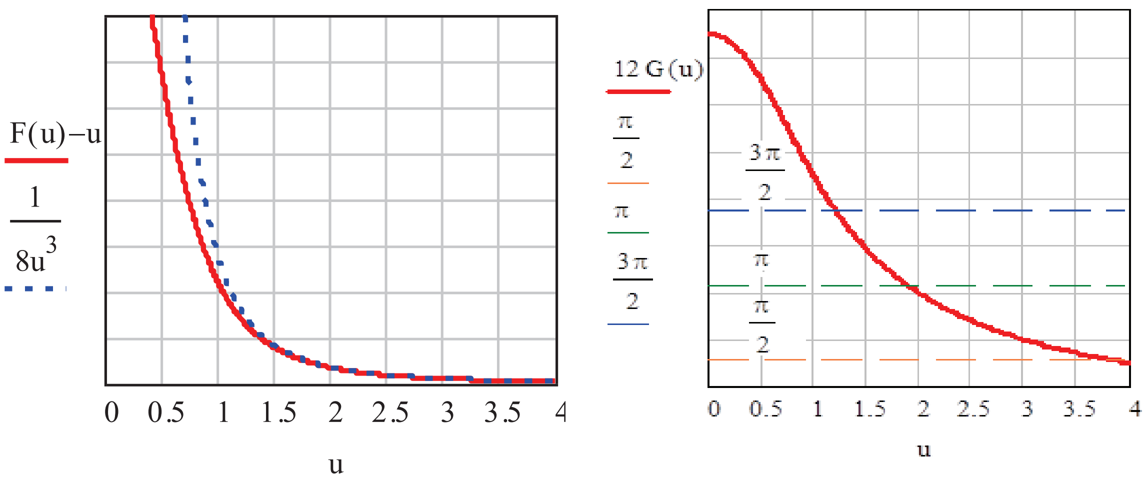

Fig. 8 - Graphical representation of F(u)-u and 12·G(u)

Based on *figures 7 and 8* an asymptotic approximations could be considered, used to formulate approximation criteria of the integral:

$$F(u)\to u,\, G(u)\to\frac{1}{2u},\, F(u)-u\to\frac{1}{(2u)^3};\quad \text{for } u>1 \tag{13}$$

The first term from equation (11) is a complex function, with real and imaginary components that are decreasing dependent on the physical parameters of the cable system. As opposed to the first mentioned term, the other three terms are depending on the cable system physical properties. Based solely on the second term, a truncation criterion for integral is proposed. By replacing the semi-infinite integral with a finite one:

$$\int_0^{\infty}\exp(-\xi u)du \cong \int_0^{u_{\max}}\exp(-\xi u)du \tag{14}$$

which leads to the relative error of the approximation:

$$\varepsilon_r=\exp(-\xi\cdot u_{\max})\Rightarrow u_{\max}=\frac{-\ln(\varepsilon_r)}{\xi} \tag{15}$$

Using empirical methods [14] it is possible to establish the value of ξ to provide $u_{max}>2$. The third term from equation (12) is a complex one, with irregular oscillations

$$\exp[-j\xi G(u)]=\cos[\xi G(u)]-j\sin[\xi G(u)] \tag{16}$$

If the condition from equation (17) is fulfilled, the last term does not oscillate:

$$\xi G(u)<\frac{\pi}{2}\Rightarrow\xi>\frac{\sqrt{2}}{2}\pi\cong 2.221 \tag{17}$$

The graphical representation of $12\cdot G(u)$ from *figure 8,* is made for a value of ξ parameter higher than the value indicated in equation (17).

Each value which is an odd multiple of $\pi/2$, corresponds to a zero crossing of the cosine term in (16), while even multiples of $\pi/2$ correspond to the zeros of the sine term. The total number of zeros is $n=\left[\frac{\sqrt{2}\xi}{\pi}\right]$ and the values of u corresponding to these zeros are obtained by the relation:

$$u_k=G^{-1}\left(\frac{k\pi}{\xi\sqrt{2}}\right)=\frac{\sqrt{\xi^4-\frac{k^4\pi^4}{4}}}{\xi k\pi},\quad k=n,\ldots,2,1 \tag{18}$$

Figure 9 shows the irregular oscillations of the third term by a set plot for $\cos[\xi G(u)]$ and $\sin[\xi G(u)]$ for different values of ξ.

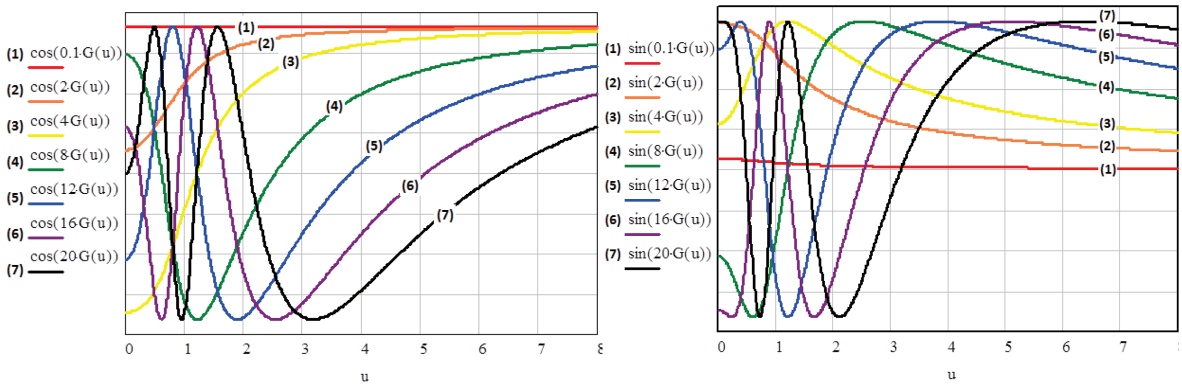

Fig. 9 - Graphical representation of cos[ξG(u)]and sin[ξG(u)]

The fourth term from equation (12) is a cosine function, so it is an oscillatory regular term. The number of zeros within the truncated interval of u (that is $[0, u_{max}]$) is:

$$z = \left[\frac{\xi \eta u_{\max}}{\pi} + \frac{1}{2} \right] \tag{19}$$

This doesn't present oscillations in the truncated interval if: $\eta < \dfrac{\pi}{2\xi u_{\max}}$.

Uribe [14] does a comprehensive study of Pollaczek's integral and points out the difficulties encountered when attempting its numerical solution. His analysis provides a strategy to avoid these difficulties. Furthermore, he provided solutions to Pollaczek's integral in graphic form for a broad range of applications besides normalized impedances for underground cables within the same broad application range. Finally, he compares the normalized impedances against those from a widely used approximation. His studies focus mainly on underground cables.

4. PROPOSED NUMERICAL INTEGRATION PROCEDURES

New analytical formulas for ground correction terms for self and mutual impedances are derived [6, 15, 16]. The determined formulas contain semi-infinite integral terms which are calculated by a novel stable and efficient numerical integration scheme in order to overcome the problems arising from the oscillate form of the infinite integrals.

In the solution of larger electromagnetic interference problems, it is often needed the evaluation of an integral that is function of a parameter. A choice of numerical integration may be given by the use of a family of orthogonal polynomials. These polynomials generate a Gaussian quadratic rule, according to the following theorem: assume a weight function on the interval *[a; b]*, respectively a family of orthogonal polynomials $\{\varphi_k(x)\}$ with respect to this weight function and this interval [6, 16].

The quadratic rule is defined by:

$$G_n(x) = \sum_{i=1}^{n} w_i^{(n)} \cdot f\left(x_i^{(n)}\right) \tag{20}$$

For $x_i(n)$ the roots of φ_n and $w_i^{(n)}$ is given by relation above:

$$w_i^{(n)} = \int_a^b w(x) \cdot \left(\prod_{k=1,\ k\neq i}^{n} \frac{x - x_k^{(n)}}{x_i^{(n)} - x_k^{(n)}} \right) dx \tag{21}$$

Then $G_n(p)$ is exact for all polynomials $p \in P_{2 \cdot n-1}$, and there exists $\xi \in [a;b]$ such that

$$\int_a^b w(x) \cdot f(x) dx - G_n(f) = \frac{1}{(2 \cdot n)!} \cdot \left(\int_a^b \Psi(x) dx \right) f^{(2 \cdot n)}(\xi_n) \tag{22}$$

For all $f \in C^{2 \cdot n}([a;b])$, where:

$$\Psi_n(x) = \prod_{k=1}^{n} \left(x - x_k^{(n)} \right)^2 \tag{23}$$

Appling this theorem to construct a Gaussian quadratic rule for semi-infinite integrals of the form leads to:

$$I(f) = \int_0^\infty e^{-x} \cdot f(x) dx \tag{24}$$

For our case, the weight function is the reduced decaying exponential $w(x) = e^{-x}$, so the orthogonal polynomial family that we need to use is the Laguerre family. We choose the solutions of the fourth-order Laguerre polynomial:

$$L_4(x) = \frac{1}{24} \cdot \left(x^4 - 16 \cdot x^3 + 72 \cdot x^2 - 96 \cdot x + 24 \right) \tag{25}$$

Then the weights are correspondingly evaluated, by the particular form of relation above:

$$w_1^{(4)}(h) = \int_0^\infty e^{-x \cdot h} \cdot \prod_{k=1,\ k\neq i}^{4} \frac{\left(x - x_k^{(4)}\right)}{\left(x_i^{(4)} - x_k^{(4)}\right)} \tag{26}$$

The parameter *h* provides an extension in the numerical evaluation of the semi-infinite integral. These weight integrals are computed using some kind of numerical integration routine, such as the trapezoid of Simpson rule. Their values, according to the imposed parameter are exposed in the following *Table 1*:

Table1. Numerical values of the weight integrals

h [m]	**1**	**2**	**3**
w_1	0.693	0.441	0.339
w_2	0.357	0.056	-0.011
w_3	0.039	0.0024	0.0055
w_4	0.0005	-0.00019	-0.00047

On these conditions, we may apply the explained solving procedure to the case of a semi-infinite integral involving the evaluation of a pipeline to ground impedance [15], and indirectly the induced potential or current density on the metallic pipeline due to the coexistence with a high voltage power grid, above the ground. The explicit integral is:

$$I = \int_0^\infty e^{y \cdot \sqrt{x^2+m^2} - x \cdot h} \cdot \frac{\cos(u \cdot x)}{x + \sqrt{x^2 + m^2}} dx \tag{27}$$

Considering the evaluation of this integral at the surface level of the earth *y=0* and the *m* coefficient depends on the value of the working frequency of the power grid, as this relation shows:

$$m = \sqrt{\frac{j \cdot 2 \cdot \pi \cdot f \cdot 4 \cdot \pi \cdot 10^{-7}}{50}} \tag{28}$$

the involved function is:

$$f(x,u) = \frac{\cos(u,x)}{x + \sqrt{x^2 + m^2}} \tag{29}$$

and it remains only to apply and evaluate the quadratic rule:

$$I(u,h) = \sum_{i=1}^{4} w_i(h) \cdot f(x_i, u) \tag{30}$$

For an imposed test parameter *u=1,2,3*, in the conditions presented above, regarding the weight integrals and the parameter *h*, a matrix of results it is achieved.

Table2. Numerical values of the semi-infinite integral – test case

h \ u	1	2	3	
1	0.868-1.681j	0.646-1.231j	0.049-0.946j	$\cdot 10^{-5}$
2	0.647-1.411j	0.531-1.036j	0.042-0.797j	
3	0.584-1.01j	0.039-0.736j	0.029-0.565j	

It is worthwhile to notice that the results are complex values, and their physical significance relates to mutual impedance, with the real part – the resistance and the imaginary part – the reactance.

It is observed that most of the area under integrand occurs for small values of x, especially when y is large. This fact suggests that we might seek approximations of integral by replacing an exponential or algebraic function, the objective being to permit analytic integration.

Since there is no good systematic method for making these replacements, their success depends directly on the intuition and ingenuity, taking into account that in practice the integrand has limited accuracy.

4. CONCLUSIONS

The paper presents a numerical evaluation of the self and mutual impedance formulas for lines with earth return, formulas which contain semi-infinite integrals with high oscillation integrand. New analytical formulas for these terms are presented in the paper. We might seek approximations of the semi-infinite integrals by replacing an exponential or algebraic function, the objective being to permit analytic integration. Since there is no good systematic method for making these replacements, their success depends directly on the intuition and ingenuity, taking into account that in practice the integrand has limited accuracy.

It can be observed that the integrand of Pollaczek infinite integral has negative real part and positive imaginary part at high frequencies, which result in negative resistance and inductance. The incorrect integrant in a high frequencies region causes the numerical instability of the infinite integral. So, it is necessary to find an applicable analytically limit for this infinite integral.

However, it is very hard to calculate the earth-return impedance, but a very promising solution to the problem might be a numerical electromagnetic analysis which not requires any impedance and admittance formulas, which are compulsory to use in circuit theory approach.

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