THE ROUTH AND HURWITZ STABILITY STUDY USING LABVIEW PROGRAMME

Barz **CRISTIAN**, Oprea **CONSTANTIN**
Technical University of Cluj-Napoca, North University Center from Baia Mare
cristian.barz@ubm.ro, prconstantin@yahoo.com

Key words: automatic systems stability, LabVIEW platform
Abstract*: Because the automatic system stability analytic study is very difficult to achieve, the paper presents some computer programmers realized in "LabVIEW" and useful for the study. The programmers were done taking into account the stability of the systems [1, 2, 3, 4] without insisting too much on the theoretical problems.*

1. THEORETICAL CONSIDERATIONS

The problem of the linear and nonlinear automatic systems stability is very complex because in the case of systems with negative reaction, there is the possibility, in certain circumstances to get out of control. This state is actually instability because the signals transmitted and processed in the system are no longer the result of the commands given. The stability of a system is first required as a designing condition and secondly as an operation condition because in many cases a simple wanted or not wanted variation of a constructive parameter can cause the system's instability. The systems that are stabile only for certain constructive parameters values are known as conditioned stable systems.

Can be declared that an automatic system is stable if, after a given command or a disturbance the emergence signal e(t) tend to a stationary value and the weight function y(t) tends to zero:

$$\lim_{t\to\infty} e(t) = e_{st.}\,;\ \lim_{t\to\infty} y(t) = 0 \tag{1}$$

the condition required for the weight function being more general because it does not suppose the existence of a command signal.

Considering a close circuit system with: Yd(s) – transfer function on the direct curl, Yr(s) – transfer function on the reaction curl, s = jω ($j = \sqrt{-1}$; ω – pulsation), the system's function is:

$$Y_o(s) = \frac{Y_d(s)}{1 + Y_d(s) \cdot Y_r(s)} \qquad (2)$$

The problem of the described (2) system's stability occurs when analyzing the poles of this function, which means the zeroes of the polinom:

$$1 + Y_d(s) \cdot Y_r(s) = 0 \qquad (3)$$

more precisely the existence or inexistence of positive zeroes or complex conjugated with positive real part zeroes.

To solve the stability systems problem were realized many stability criterions such as: Routh and Hurwitz; Liapunov; Lienard-Chipard; Bode (in logarithmic diagrams); etc.

The analytic solution for this stability study using these criterions is very laborious. That's why in this paper is offered a solution by expanding "LabVIEW" programmer area.

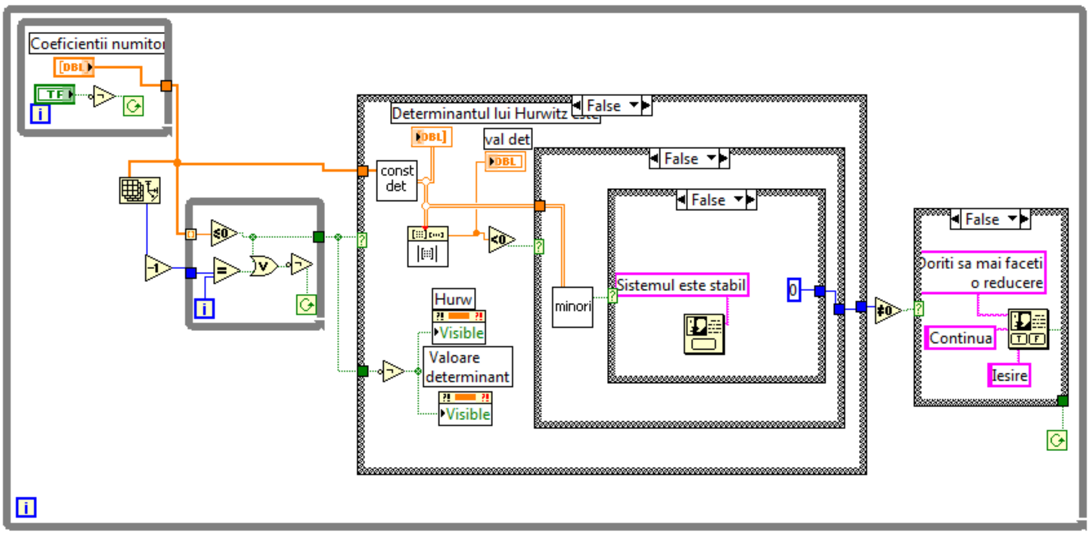

Fig 1.1. Routh – Hurwitz stability criterion (algebraically)

Even if many papers present them separately, both have the same essence, being algebraic criterions that analyses the stable asymptotic character of a system by determining the inexistence of positive real part of the characteristic polinomous (3), but without actually finding these roots.

We are considering an autonom linear system with the following matrices' equation:

$$\mathbf{x}^T(t) = \mathbf{A} * \mathbf{x}(t) \qquad t\ (4)$$

where $\mathbf{x}^T = (x_1, x_2, \ldots x_n)$ – state vector; A – coefficients matrices, constant, nonsingular and with definite (n*m) elements. The system described by such an equation is asymptotic stable, if and only if all the values belonging to A are at the same time the roots of the characteristic polinom:

$$det\,[s\mathbf{I} - \mathbf{A}] = a_{0*}s^n + a_{1*}s^{n-1} + \ldots\ldots a_{n-1*}s + a_n \qquad (5)$$

where s = d/dt is the derivation operator and **I** is the unit matrices.

The polinom who's roots are negative or with negative real part is known as *Hurwitz polinom*.

Results that, if the characteristic polinom of the analyses system is a Hurwitz polinom the studied system is asymptotic stable.

One of the conditions required, but not sufficient for a polinom to be Hurwitz polinom is all the coefficients to be positive and different from zero:

$$a_1/a_0 > 0;\ a_2/a_0 > 0;\ \ldots\ldots a_n/a_0 > 0 \qquad (6)$$

$$\left|\begin{array}{ccccc} a_0 & a_2 & a_4\ \ldots\ldots a_{n-2} & & a_n \\ a_1 & a_3 & a_5\ \ldots\ldots 0 & & 0 \\ b_1 & b_2 & b_3\ \ldots\ldots 0 & & 0 \\ c_1 & c_2 & c_3\ \ldots\ldots 0 & & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ d_1 & d_2 & 0\ \ldots\ldots 0 & & 0 \\ e_1 & e_2 & 0\ \ldots\ldots 0 & & 0 \\ f_1 & 0 & 0\ \ldots\ldots 0 & & 0 \\ g_1 & 0 & 0\ \ldots\ldots 0 & & 0 \end{array}\right| \qquad (7)$$

condition that can be verified immediately [3].

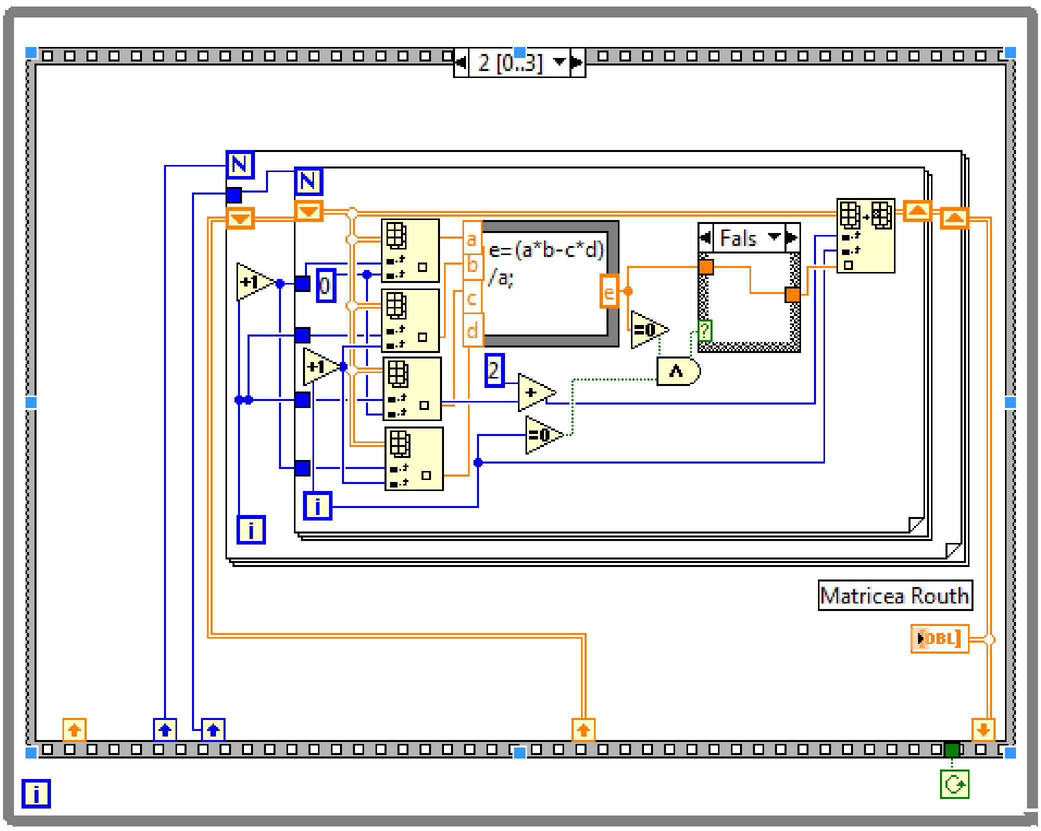

Fig 1.2. The Routh matrice

In Routh's formulation, using the polinom coefficients [5] is obtained the following table:
where the first two lines are formed by the polinom coefficients (5), the followings are calculated with the expressions:

$$b_1 = \frac{a_1 \cdot a_2 - a_0 \cdot a_3}{a_1}; b_2 = \frac{a_1 \cdot a_4 - a_0 \cdot a_3}{a_1}; b_3 = \frac{a_1 \cdot a_6 - a_0 \cdot a_7}{a_1} \ldots.$$
$$c_1 = \frac{b_1 \cdot a_3 - b_2 \cdot a_1}{b_1}; c_2 = \frac{b_1 \cdot a_5 - b_3 \cdot a_1}{b_1}; c_3 = \frac{b_1 \cdot a_7 - b_4 \cdot a_1}{b_1} \ldots. \quad (8)$$

and the last two lines are:

$$f_1 = \frac{e_1 \cdot d_2 - d_1 \cdot e_2}{e_1}; \quad g_1 = e_2 \quad (9)$$

Totally will be n+1 lines, two successive lines having the same numbers of terms different from zero, excepting the second line which can have a term less than the first one, in case that the polinom degree is odd (the presented case).

The polinom (5) is a Hurwitz polinom and results that the system characterized by it is asymptotic stable if the inequalities (6) are fulfilled and also all the terms of the first column in (7) are positive:

$$a_0 > 0; a_1 > 0; b_1 > 0; \ldots\ldots f_1 > 0; g_1 > 0 \quad (10)$$

In case condition (10) is not fulfilled, results that the polinom (5) is not a Hurwitz polinom, the table (7) showing us the number of the positive roots or the roots with positive real part of the polinom, number equal with the number of the sign changes that appear in the first column of the table.

2. CONCLUSIONS

Depending on the stability criterion suitable were realized programmers in "Lab VIEW"[5,6] and based on concrete systems examples with analytic verified stability the programmers were used. The results obtained were very good, the advantage of using these programmers being the work time that can be considered infinitesimal compared with the time necessary for analytic solving and the work accuracy that can't be expressed. With the mention that the programmers were made for the stability criterions: Routh-Hurwitz; Lienard-Chipard; Liapunov; Nyquist; Bode(in logarithmic diagrams that can be success-fully also used for automatic not linear systems based on the description function), the theoretic aspect of the other criterions are not presented in the paper. Are recommended [1,2,3,4].

REFERENCES

1. Hăngănuţ M., *Automatică,* Editura Didactică şi Pedagogică, Bucureşti, 1971.

2. Hăngănuţ M., Dancea I., Negru O., *Programe FORTRAN comentate în automatică,* Editura Tehnică, Bucureşti, 1974.

3. Oprea C., *Automatică,* Editura Risoprint, Cluj Napoca, 2000.

4. Oprea, C., Barz, Cr., *Elemente de inginerie electrica, reglare automata si automatizari,* Risoprint, Cluj Napoca, 2011.

5. *** *LabVIEW, User Manual*, Natonal Instrument, USA, 2010.

6. *** *LabVIEW, Programming, Reference Manual,* Natonal Instrument, USA, 2010.