A NEW FINITE ELEMENT METHOD DISCRETIZATION BASED ON DUAL FORMULATION OF MAGNETOSTATIC FIELD

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Abstract: The meshing techniques are very important for the economy and the precision of a Finite Eelment Method (FEM) analysis. The presented algorithm for the discretization of a magnetostatic plane parallel configuration is based on dual formulation of magnetostatic field and generates, in iterative steps, a final mesh which is very close to the magnetostatic spectrum, magnetic field lines and equipotentials lines for scalar magnetic potential.

1. INTRODUCTION

Discretization of the problem region is the most delicate step, the bigger error generator of an FEM field analysis. Using triangular first order elements, with a careful meshing, the solutions have good accuracy and the mathematical problem becomes simple. Generated particularities, offering the solution as joined uniform fields with fixed geometrical position established in preprocessor by discretization, rarely gives an acceptable ratio: accuracy, number of nodes. So the mesh refinement techniques, permanently adapted to the level of error, are a compulsory step in modern FEM. Proposed algorithm goes to obtain the best solution with the initially imposed number of nodes without increasing the polynomial order of elements. Also it offers the possibilities to reduce the studied domain and to improve the domain frontiers and/or the associated conditions if there are imposed using truncation methods [3]. Dual formulations of the magnetostatic fields generate simplicity, a very strong error estimator, not needing numbering techniques and composing the system matrix, etc.

2. DUAL FORMULATION OF MAGNETOSTATIC FIELDS

In current-free regions magnetostatic simulation implies simultaneously solving the equations,

$$\nabla \overline{B} = 0 \tag{1}$$

$$\nabla \times \overline{H} = 0 \tag{2}$$

$$\overline{B} = \mu_0 \mu_r (H) \cdot \overline{H} \tag{3}$$

associated with boundary conditions at the interface Γ between media 1 and media 2,

$$\overline{n} \times \left(\overline{H_1} - \overline{H_2}\right)\Big|_{\Gamma} = 0 \tag{4}$$

$$\overline{n} \cdot (\overline{B_1} - \overline{B_2})\Big|_{\Gamma} = 0 \tag{5}$$

Introducing both magnetic potentials, vector and scalar,

$$\overline{B} = rot\,\overline{A} \tag{6}$$

$$\overline{H} = -\nabla V_m \tag{7}$$

two mathematical formulations are available:

- Electrical where (1), (3), (5) are verified exactly, the errors are in (2), (4) that is magnetic vector potential formulation,
- Magnetical where (2), (3), (4) are verified exactly, the errors are in (1), (5) that is magnetic scalar potential formulation.

3. FEED BACK CRITERIA

The success of a mesh adaption algorithm is strongly dependent on the feed back criteria. Starting from the dual formulation of the magnetostatic fields it is easy to estimate the error of the analysis using error in constitutive relation [2] that has the main advantage to be the upper bound of the exact error. With $\overline{B}$ and $\overline{H}'$, the FEM solutions obtained from vector, respectively scalar magnetic potentials,

$$e = \overline{B} - \mu \overline{H}' \tag{10}$$

The global absolute and relative error associated are defined in a bounded domain D,

$$e_{abs}^2 = \frac{1}{2} \int_D \frac{1}{\mu} \left(\overline{B} - \mu \overline{H}'\right)^2 dD \tag{11}$$

$$\varepsilon^2 = \frac{\int_D \frac{1}{\mu}\left(\overline{B} - \mu H'\right)^2 dD}{\int_D \frac{1}{\mu}\left(\overline{B} + \mu \overline{H'}\right)^2 dD} \tag{12}$$

4. SPECTRUM TYPE DISCRETIZATION

An imputed initial mesh, generator of the field geometry and the maximum number of nodes, start a typically dual FEM analysis. The solution of the potentials (vectorial and scalar) allows to calculate the position of intersections between flux lines and equipotentials, solving the equations system:

$$\begin{cases} V_{imp}(k_2) = a + b \cdot x(k_1, k_2) + c \cdot y(k_1, k_2) \\ A_{imp}(k_1) = a_1 + b_1 \cdot x(k_1, k_2) + c_1 \cdot y(k_1, k_2) \end{cases} \tag{13}$$

with V_{imp} and A_{imp} the scalar and vector potentials choused to generate the field spectrum and constants *a, b, c, a_1, b_1, c_1* computed from the "old" coordinates of the nodes and corresponding values of the potentials:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{pmatrix}^{-1} \begin{pmatrix} V_{m1} \\ V_{m2} \\ V_{m3} \end{pmatrix} \tag{14}$$

Replacing V_m with *A,* we obtain the constants for vector potentials equations.

The cycle is repeated until the imposed error for geometrical displacement of the mesh is reached.

For the e element, see fig. 1.a, based on FEM calculation of the magnetic flux density and field:

$$gradA = \bar{i}\frac{\partial A}{\partial x} + \bar{j}\frac{\partial A}{\partial y} = \frac{1}{2S}\left[(y_3 - y_2)\bar{i} - (x_3 - x_2)\bar{j}\right](A_2 - A_1) \tag{15}$$

$$gradV_m = \bar{i}\frac{\partial V_m}{\partial x} + \bar{j}\frac{\partial V_m}{\partial y} = \frac{1}{2S}\left[(y_3 - y_1)\bar{i} - (x_3 - x_1)\bar{j}\right](V_{m2} - V_{m1}) \tag{16}$$

$$\overline{H'} = \frac{1}{2S}\left[-(y_3 - y_1)\bar{i} + (x_3 - x_1)\bar{j}\right](V_{m2} - V_{m1}) = \frac{1}{2S}(V_{m2} - V_{m1})l_{13}\overline{n_{13}} \tag{17}$$

$$\overline{B} = -\frac{1}{2S}\left[(x_3 - x_2)\bar{i} + (y_3 - y_2)\bar{j}\right](A_2 - A_1) = \frac{1}{2S}(A_2 - A_1)\bar{l}_{32} \tag{18}$$

with simple geometrical calculation it is easy to see that the interelemental boundary conditions are satisfied with the position precision.

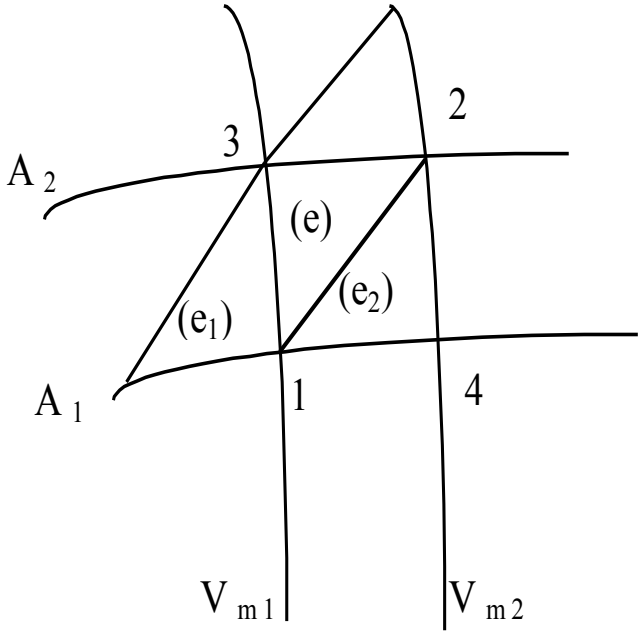

Fig.1. a. An element of the spectrum type discretization

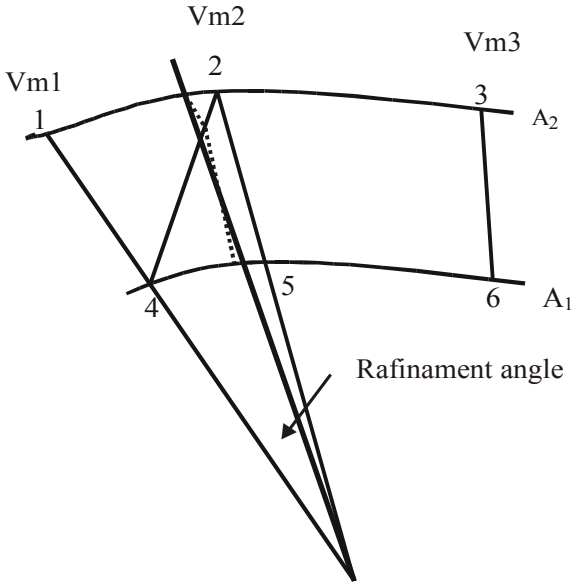

Fig.1. b. Refinement of the equipotential line 25

The feed back criteria, for each element, is reached if the error in constitutive relation become scalar and:

$$e_{(e)} = B - \mu H' = \frac{1}{2S}\left(l_{23}\left|A_2 - A_1\right| - \mu l_{13}\left|V_{m_2} - V_{m_1}\right|\right) = 0 \text{ or } \frac{l_{23}}{l_{13}} = \mu\left|\frac{V_{m_2} - V_{m_1}}{A_2 - A_1}\right| \tag{19}$$

Satisfying both conditions goes to rectangular spectrum cells. With exception of the uniform field the zero error could not be touched.

The presented algorithm improved the analysis precision by transforming the spectrum cells in "most appropriate rectangles" choosing the best values of potentials V_{imp} and A_{imp}, given by the two refinement angles and two refinement directions.

The refinement directions have to be choused as the areas with most unaligned flux lines between two equipotentials and most unaligned equipotentials between two flux lines.

The calculus of the refinement angle is based on the determination of the arithmetical media of the same type of lines angles followed by the refinement directions. With these values there are computed the desired values to be imposed on potentials as shown in fig.1.b.. The position of line 25 is redrawn at the same angle to both neighborly lines 14, 36. The value for potential can belong to any point of intersection between the dotted line (FEM line in those two elements) and the frontiers. The last step of fitting the discretization trough the spectrum is recommended to validate the discretization.

A control of the boundary conditions becomes possible passing the quality of being the frontier to a close line of the spectrum. Choosing the new frontier as a far distance line or infinitely close one the analysis area will be decrease or not.

The entire algorithm computing schemes are based on recursive strategies given by the first impute of the mesh, made in a matrix disposal of nodes as intersections of equipotentials and flux lines. No numbering technique is necessary and the system matrix is not defined.

5. EXAMPLE

An infinite ferromagnetic (infinite relative magnetic permeability) wire is placed in a uniform magnetic field perpendicular to its filed lines. An initial 2162 nodes mesh, fig. 3.b, sets the geometry, fig.3.a.

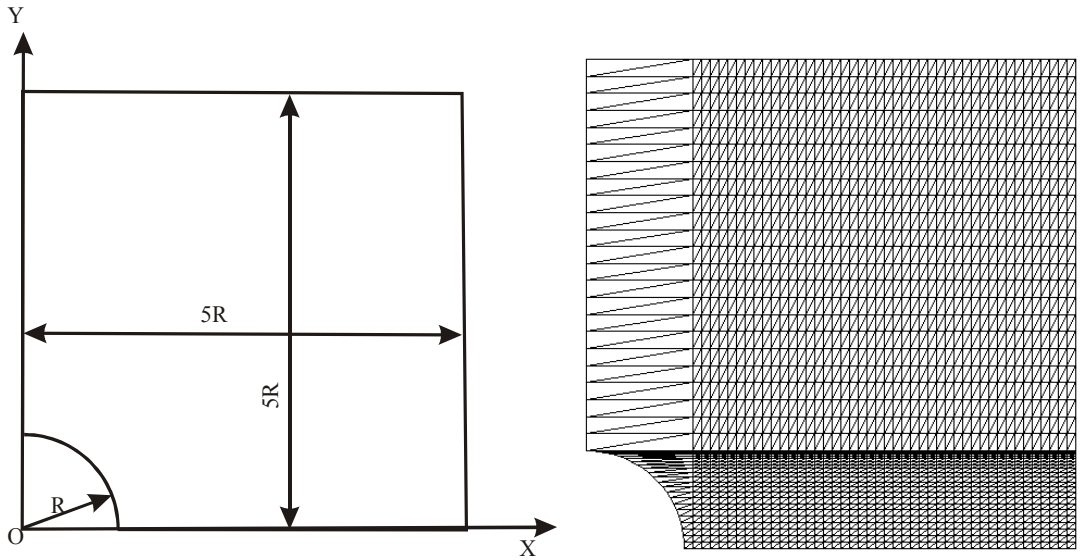

Fig.3.a. The geometry of the analysis domain; b. The 2162 initial mesh

The refined mesh, witch also performed the decreasing of the analysis domain to a quarter of started one, shown in Fig. 4.a, generate the magnetic spectrum, Fig.4.b and reached the global relative error 1.53% in tree steps.

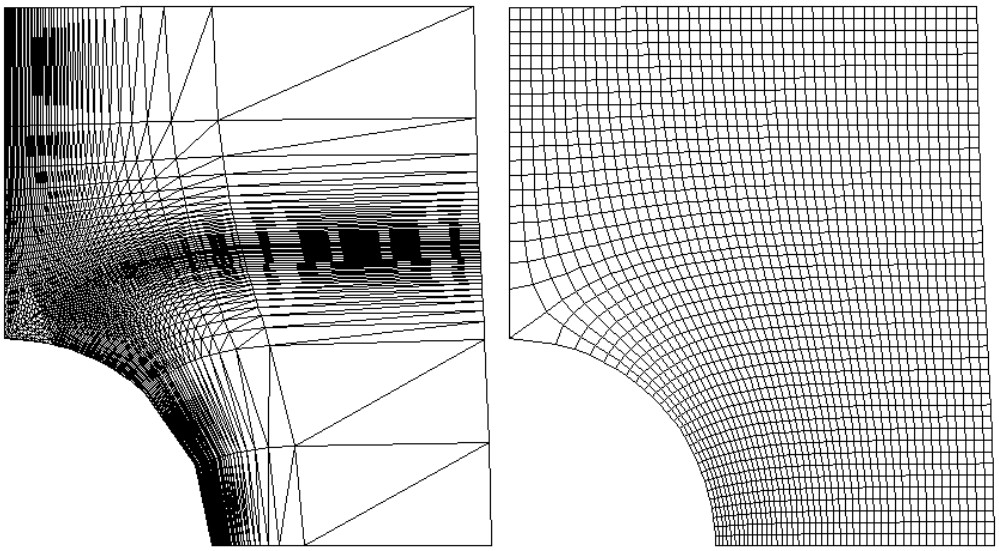

Fig.4. a. Refined mesh; b. magnetic spectrum

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