

## ON DISTRIBUTED MODEL PREDICTIVE CONTROL FOR LOAD FREQUENCY PROBLEM

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**Abstract:** *The paper describe a multi-agent application in power systems for the problem of Load Frequency Control. The connections between subsystems are treated by each controller agent as a set of disturbance signals. Each area maintain the tie-lines power flow to specified values, based on communication between neighboring agents.*

### 1. INTRODUCTION

One of the most implemented advanced control techniques in last decade in the process industries [1] is Model Predictive Control (MPC), and its popularity is due to the versatility in coping with constraints. The main concept of MPC is to use a model of the plant to predict the future evolution of the system and is based on the idea of finite receding horizon, emulating infinite horizon optimal control algorithms.

Using a model of the system to be controlled, at each sample period  $t$  an optimal control problem is solved with the aid of constrained numerical optimization methods. Following this, only the first part of the solution is implemented for the duration of the sample period. Due to model uncertainty and disturbances, the actual output trajectory may deviate from the predicted trajectory, thus a measurement of the actual output at the next sample instant  $t + 1$  is taken, and the optimal control problem is updated with the new measurement. This process of measuring, solving a constrained optimization problem and implementing only the first part of the optimal control sequence is repeated at future sample instants, and in this way a feedback control law is produced. A thorough survey on the subject is [2].

From an algorithmic perspective, most MPC implementations result in the requirement to solve, at each sample instant, a quadratic program (QP) :

$$\min_x \frac{1}{2} x^T H x + f^T x, \quad s. t. \begin{cases} A \cdot x \leq b \\ A_{eq} x = b_{eq} \\ LB \leq x \leq UB \end{cases} \quad (1)$$

This is a centralized approach that is considered impractical for the control problem of large-scale systems (such as power networks), when the optimization problem is too big for real time computation. The solution may be to decompose the problem into a set of smaller subproblems and the overall system into appropriately subsystems with distinct MPC controllers for each subsystem.

The new method of distributed MPC can be solved in a parallel manner if the controllers are well coordinated; we intend to realize this by a particular communication among the agents and not through a centralized supervisor. Thus, each subsystem problem will be solved by an individual controller agent using local information and collaborating to other agents to achieve global decisions.

Multi-agent systems (MAS) paradigm has matured during the last decade and effective applications have been used; in MAS tasks are deploy by interacting entities (abstraction objects named *agents*), capable of autonomous actions in its environment; agents cooperates with each other, but each agent has incomplete information or capabilities for solving the problem (has a limited viewpoint); there is no system global control, data are decentralized and computation is asynchronous [3].

In industrial application agent technology can be use in process automation functions where the tasks require cooperative distributed problem solving. Typical applications refer to cooperative robots, sensor networks, traffic control, electronic markets. MAS can be considered "self-organized systems" as they tend to find the best solution for their problems without external intervention. Multi-agent technologies can be applied also, in a variety of applications related to power system, such as disturbance diagnosis, restoration, secondary voltage control or power system visualization [4].

## 2. MULTI-AGENT MODEL PREDICTIVE CONTROL

We will consider a network that is partitioned into  $n$  subnetworks and each subsystem model is represented as a discrete, linear time-invariant (LTI) model of the form (3)

$$\begin{cases} x_i(k+1) = A_i x(k) + B_i u_i(k) + E_i d_i(k) + H_i w_i(k) \\ y_i(k) = C_i x(k) \end{cases} \quad (2)$$

where at time  $k$  for subsystem  $i$ ,  $x_i(k) \in \mathbb{R}^{n_{x_i}}$  are local states,  $u_i(k) \in \mathbb{R}^{n_{u_i}}$  are the local inputs,  $d_i(k)$  are the local known perturbation,  $y_i(k) \in \mathbb{R}^{n_{y_i}}$  are the local outputs and  $w_i(k)$  are external influences due to interconnections between subsystems.

For each subsystem the controller will be implemented by a software agent; in each step  $k$  the agent compute the next command  $u_k$  solving an optimization problem (4) by collaboration with other similar agents.

$$J_{i,k} = \frac{1}{2} \sum_{p=0}^{N-1} \left[ \|x_{k+p+1}\|_{Q_x}^2 + \|\Delta u_{k+p}\|_{Q_u}^2 \right] \quad (3)$$

The expression of objective function from equation (4) can be expanded in order to reformulate the MPC problem such as a quadratic programming (QP) problem for which solvers are easy to find.

Just for simplicity will consider in next equations a prediction horizon  $N=3$

$$\begin{aligned} J_i &= \frac{1}{2} \sum_{k=0}^{N-1} \left[ \|Ax_k + Bu_k + Hw_k\|_{Q_x}^2 + \|u_k - u_{k-1}\|_{Q_u}^2 \right] = \\ &= \frac{1}{2} \left[ \|Ax_0 + Bu_0 + Hw_0\|_{Q_x}^2 + \|A^2x_0 + ABu_0 + AHw_0 + Bu_1 + Hw_1\|_{Q_x}^2 + \right. \\ &\quad \left. + \|A^3x_0 + A^2Bu_0 + A^2Hw_0 + Bu_2 + Hw_2\|_{Q_x}^2 + \right. \\ &\quad \left. + \|u_0 - u_{-1}\|_{Q_u}^2 + \|u_1 - u_0\|_{Q_u}^2 + \|u_2 - u_1\|_{Q_u}^2 \right] \\ &= \frac{1}{2} \left( \begin{bmatrix} A \\ A^2 \\ A^3 \end{bmatrix} x_0 + \begin{bmatrix} B & 0 & 0 & \vdots & H & 0 & 0 \\ AB & B & 0 & \vdots & AH & H & 0 \\ A^2B & AB & B & \vdots & A^2H & AH & H \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ w_0 \\ w_1 \\ w_2 \end{bmatrix} \right)^T \cdot Q_x (\dots) \\ &\quad + \frac{1}{2} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ w_0 \\ w_1 \\ w_2 \end{bmatrix}^T \begin{bmatrix} 2Q_u & -Q_u & 0 & \vdots & \\ -Q_u & 2Q_u & -Q_u & \vdots & 0 \\ 0 & -2Q_u & +Q_u & \vdots & \\ \dots & \dots & \dots & \vdots & \dots \\ 0 & & & \vdots & 0 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ w_0 \\ w_1 \\ w_2 \end{bmatrix} + [-Q_u \cdot u_{-1}] \cdot u_0 \quad (4) \end{aligned}$$

and with following notations

$$\begin{aligned} \hat{A} &= \begin{bmatrix} A^1 \\ A^2 \\ A^3 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B & 0 & 0 & \cdot & H & 0 & 0 \\ AB & B & 0 & \cdot & AH & H & 0 \\ A^2B & AB & B & \cdot & A^2H & AH & H \end{bmatrix}, \\ Z &= \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ w_0 \\ w_1 \\ w_2 \end{bmatrix}, \quad H_u = \begin{bmatrix} 2Q_u & -Q_u & 0 & \vdots & \\ -Q_u & 2Q_u & -Q & \vdots & \\ 0 & -2Q & +Q & \vdots & 0 \\ \dots & \dots & \dots & \vdots & \dots \\ 0 & & & \vdots & 0 \end{bmatrix}, \quad (5) \end{aligned}$$

The weight matrix  $Q_u$ ,  $Q_u$  from (3) is given by

$$Q_u = \begin{bmatrix} q_u & 0 & \cdots & 0 \\ 0 & q_u & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & q_u \end{bmatrix}, Q_x = \begin{bmatrix} q_x & 0 & \cdots & 0 \\ 0 & q_x & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & q_x \end{bmatrix} \quad (6)$$

and the objective function of agent  $i$  in step  $k$  can be written now as a QP problem:

$$J_{i,k} = \frac{1}{2} \cdot Z^T \cdot \tilde{H} \cdot Z + f^T \cdot Z \quad (7)$$

The MPC problem in this form is equivalent to (4) for which standard and efficient codes exist, many suppliers of MPC writing their own solvers [5]. The QP solver shipped together with Matlab (*quadprog*) computes the answer with ten degrees of freedom or more in well under a second, but generally are considered rather slow in terms of computational speed; however Matlab provides a unified interface to other solvers.

The objective function (3) or (7) uses a quadratic cost function subject to the constraints imposed on the manipulated variables as well as state or/and output variables, expressing as a variable rate change or to keep the variable within certain bounds. According to system equation (2), constraints can be formulated:

$$\begin{cases} y_i^{\min} \leq y(k+i|k) \leq y_i^{\max} \\ u_i^{\min} \leq u(k+p-1|k) \leq u_i^{\max} \\ \Delta u_i^{\min} \leq \Delta u(k+p-1|k) \leq \Delta u_i^{\max} \end{cases} \quad (8)$$

but it can be easily reformulated in term of equation (7).

The communication between agents can improve the predictions about the future evolutions of interconnections variables. The observations in [6] suggest that information exchange between neighboring agents can have a beneficial effect in stability, when it leads to reduced prediction mismatch. As each system converges to its equilibrium, predictions on the behavior of neighbors should get more and more accurate to satisfy the stability condition.

Each of the agents in the system can use MPC and through an iterative scheme determine following actions, performing in parallel:

- 1) At sampling time instant  $k$ , agent  $i$  make a measurement of the current state of the subsystem  $x_k^i$ , send to and receive information from other neighboring agents.
- 2) Determine the best future behavior of local system according to a specified local objective, solving an optimization problem over a certain horizon. During this optimization there may be also communication with other agents.
- 3) Implement the first input  $u_k^i$  of found actions until the next step.
- 4) Move horizon to the next sampling time. Move on to the next decision step.

### 3. MULTI-AREA POWER SYSTEM MODEL

The frequency is one of the main variables characterizing the power systems. The purpose of load-frequency control (LFC) is to keep power generation equal to power consumption under consumption disturbances, such that the frequency is maintained close to a nominal frequency. LFC is becoming much more significant today due to deregulation of power systems, and in last years a number of decentralized control strategies has been developed for load-frequency control [7].

In a distributed manner, we consider more interconnected power subsystems where each area must contribute to absorb any load change such that frequency does not deviate and also, must maintain the tie-line power flow to its pre-specified value. If each considered area is supervised by an controller agent, the agents have to obtain agreement with other agents on power flowing over lines between subnetworks in order to be able to perform adequate local frequency control.

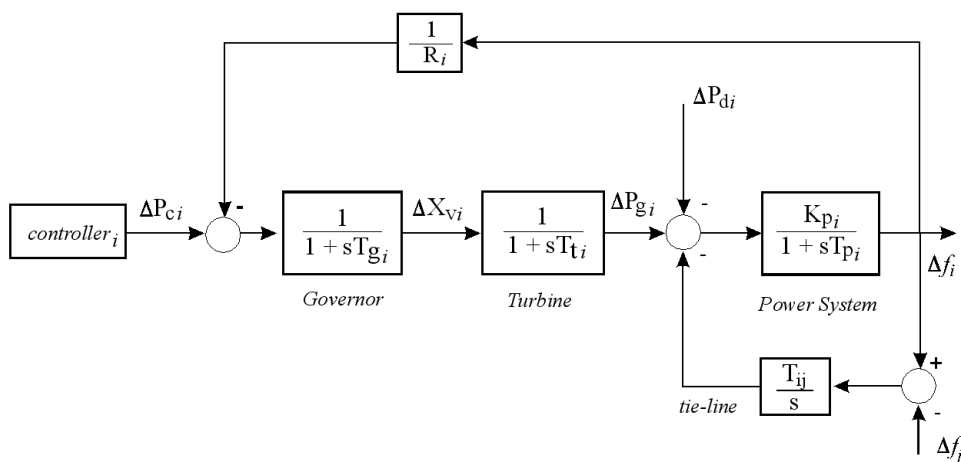


Fig.1 – Diagram for the subsystem *i* of multi-area system

Models for electric power systems are generally nonlinear. However, for load frequency control, the linearized model is generally used to design control schemes. Similar to [7][8][9], Fig. 1 shows a block diagram for the *i*th subsystem of a multi-area power system.

The notations used in the dynamic model description of the *i*th area power subsystem  $i \in \{1, \dots, n\}$  are as follows:

- $\Delta f_i$  incremental change in frequency (Hz)
- $\Delta\Delta\delta_i$  change in rotor angle
- $\Delta P_{gi}$  incremental change in generator output (p.u. MW)
- $\Delta X_{vi}$  incremental change in governor valve position (p.u. MW)
- $\Delta P_{ci}$  incremental change in integral control
- $\Delta P_{ti}$  incremental change in the tie line power (p.u. MW)

- $\Delta P_{di}$  load disturbance (p.u. MW)
- $T_{gi}$  governor time constant (s)
- $T_{ti}$  turbine time constant (s)
- $T_{pi}$  plant model time constant (s)
- $K_{pi}$  plant gain for  $i$ th area subsystem
- $R_i$  speed regulation due to governor action (Hz /p.u. MW)

The state variable equations from block diagram are derived as follows [8],[9]:

$$\dot{\Delta\delta}_i(t) = 2\pi\Delta f_i \tag{9}$$

The differential equation for the power system:

$$\dot{\Delta f}_i(t) = -\frac{1}{T_{pi}} \cdot \Delta f_i + \frac{K_{pi}}{T_{pi}} [-\Delta P_{tie} - \Delta P_{di} + \Delta P_{gi}] \tag{10}$$

$$\dot{\Delta P}_{gi}(t) = -\frac{1}{T_{ti}} \cdot \Delta P_{gi} + \frac{1}{T_{ti}} \cdot \Delta x_{vi} \tag{11}$$

The differential equation for the speed governor :

$$\dot{\Delta x}_{vi}(t) = -\frac{1}{T_{gi}} \cdot \Delta x_{vi} - \frac{1}{T_{gi} \cdot R_i} \cdot \Delta f_i + \frac{1}{T_{gi}} \cdot \Delta P_{ci} \tag{12}$$

$$\dot{\Delta P}_{tie}(t) = 2\pi \sum_{j \neq i} T_{ij} (\Delta f_i - \Delta f_j) \tag{13}$$

With state space equation, similar to (2):

$$\begin{bmatrix} \dot{\Delta\delta}_i \\ \dot{\Delta f}_i \\ \dot{\Delta P}_{gi} \\ \dot{\Delta x}_{vi} \\ \dot{\Delta P}_{di} \\ \dot{\Delta\delta}_j \end{bmatrix} = \begin{bmatrix} 0 & 2\pi & 0 & 0 & 0 & 0 \\ -\frac{K_{pi}}{T_{pi}} T_{ij} & -\frac{1}{T_{pi}} & \frac{K_{pi}}{T_{pi}} & 0 & -\frac{K_{pi}}{T_{pi}} & \frac{K_{pi}}{T_{pi}} T_{ij} \\ 0 & 0 & -\frac{1}{T_t} & \frac{1}{T_t} & 0 & 0 \\ 0 & -\frac{1}{T_{gi} R_i} & 0 & -\frac{1}{T_{gi}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \Delta f_i \\ \Delta P_{gi} \\ \Delta x_{vi} \\ \Delta P_{di} \\ \Delta\delta_j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{gi}} \\ 0 \\ 0 \end{bmatrix} \Delta P_{ci} \tag{14}$$

#### 4. SIMULATIONS

The proposed adaptive control scheme is applied to the load frequency control problem of a two subsystems. The system is simulated in discrete time steps using MATLAB programming language.

The parameters of the power systems are such that:  $T_p=20$ ,  $T_t=0.5$ ,  $T_g=0.4$ ,  $K_p=100$ ,  $R=2.7$ . At first, with  $Q_u$  selected randomly, although the outputs  $y_1$ ,  $y_2$  were stable, the interconnections variables  $w_1$ ,  $w_2$  (in fact rotor angles – first column in Fig.2) was unstable. After several simulations, we can achieve better results for  $q_u=100$ . Two independent perturbations are considered in each subsystem at time 3s and 7s respectively; load

disturbance parameter:  $\Delta P_{D,1} = 0.17$  pu and  $\Delta P_{D,2} = 0.08$  pu MW; each perturbation influences the other neighboring subsystem, as it can be seen in Fig.2.

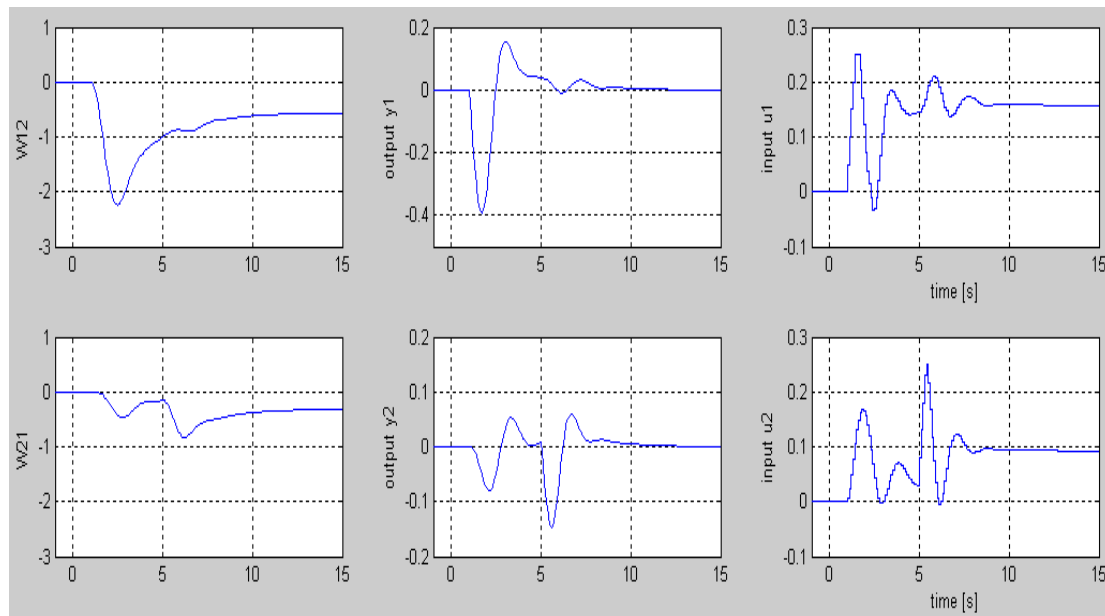


Fig. 2 – Interconnected subsystems dynamics

Figure 2 shows the evolution of the frequency deviations in each subsystem after local disturbances and also, the influence in neighboring system. With control agents the dynamics of each area become stable; for each step the inputs values are beyond imposed limits.

## 5. CONCLUSIONS

In this paper we have studied a multi-agent control application in power systems for the solution of Load Frequency Control. The tie-lines power flow are maintained to specified values based on communication between neighboring agents. The connections between subsystems are treated by each controller agent as a set of disturbance signals; they improve the predictions about the future evolutions of interconnections variables solving an optimization problem related to the paradigm of model-based predictive control.

Assuming that communication are reliable, the numerical example shows that it is possible to realise such distributed control method, based on autonomous agent behaviors.

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